Suppose we have dominos of strings, e.g.:

\[
\begin{array}{cccc}
  b & a & ca & abc \\
  ca & ab & a & c \\
\end{array}
\]

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?

T-recognizable
- do a BFS through the tree of possible sequences
Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1,b_1), (t_2,b_2), \ldots, t_k,b_k \}$$

A match is a sequence $i_1,i_2,\ldots,i_s$, where $t_{i_1}t_{i_2}\ldots t_{i_s} = b_{i_1}b_{i_2}\ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable.
First, we’ll consider MPCP where we are looking for instances that have a match that starts with the first domino.

\[ \text{MPCP} = \{ \langle P \rangle \mid P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \} \} \text{ is a PCP that has match starting with } (t_1, b_1) \} \]

**Claim**: PCP is equivalent to MPCP.
Thm 5.15: PCP is undecidable.

We'll reduce Art to PCP

Input M, w

decide Art

create PCP instance as below

we'll create dominoes:

1st:

<table>
<thead>
<tr>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>#q0 wW#</td>
</tr>
</tbody>
</table>

for:

<table>
<thead>
<tr>
<th>q0</th>
<th>u-&gt;u,R</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td></td>
</tr>
</tbody>
</table>

also:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>*</th>
</tr>
</thead>
</table>

etc. also need: dominoes for every transition of M
to be able to create new w at the end of the config.
to match top to bottom if accept appears at the bottom

Example:

M: q0 u->u,R q1 w->w,R

w = aq

# q0 w a a a #

Idea:

the PCP's solution is the config. sequence when M comp. on w
Thm: Ambiguity of CFGs is undecidable.

Example:

\[
\begin{array}{c|c|c}
\text{a} & \text{aaaab} & \text{a}\\
\text{aa} & \text{aab} & \\
\end{array}
\]

create:
\[
S \to T | B
T \to aT*; | aaaabT*; | a*; | aaaab*;
B \to aaB*; | aabB*; | a*; | aab*;
\]

now: PCP has a match iff G is ambig.