Reductions: if we can reduce (transform) problem A into a problem B, then solving problem B gives solution to problem A.

Example: $\text{HALT}_{\text{TM}} = \{ <M, w> \mid M \text{ is a TM that halts on } w \}$

**Thm 5.1:** $\text{HALT}_{\text{TM}}$ is undecidable.

**Note:** $\text{HALT}_{\text{TM}}$ is the halting problem, $\text{A}_{\text{TM}}$ is the acceptance problem.
Thm 5.1: $E_{TM}$ is undecidable, where

$$E_{TM} = \{ <M> | M \text{ is a TM and } L(M) = \emptyset \}$$

We'll reduce $A_{TM}$ to $E_{TM}$.

If we had such a green TM decider, we would be able to decide $A_{TM}$. By contradiction, $A_{TM}$ is undecidable.

$\Rightarrow$ green box cannot exist!
Thm 5.3: REGULAR\textsubscript{TM} is undecidable, where

\[ \text{REGULAR}\textsubscript{TM} = \{ <M> \mid M \text{ is a TM and } L(M) \text{ is regular} \} \]

We will reduce \( A_{TM} \) to \( \text{REGULAR}_{TM} \):

1. If input of the form \( a^n b^n \) for some \( n \geq 0 \), then input \( \epsilon \) and run \( M \) on \( \epsilon \).
2. If input not of the form \( a^n b^n \), reject.

Thus, if we have a TM decider for \( \text{REGULAR}_{TM} \), then have a TM decider for \( A_{TM} \), contradiction.

\( \uparrow \) Undecidable
Thm 5.4: \( \text{EQ}_{\text{TM}} \) is undecidable, where

\[
\text{EQ}_{\text{TM}} = \{ <M_1,M_2> \mid M_1, M_2 \text{ are TM's and } L(M_1) = L(M_2) \}
\]
Thm 5.4: $\textit{ALL}_{\textit{CFG}}$ is undecidable, where

$$\textit{ALL}_{\textit{CFG}} = \{ <G> \mid G \text{ is a CFG and } L(G) = \Sigma^* \}$$