Post Correspondence Problem

Suppose we have dominos of strings, e.g.:

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>a</th>
<th>ca</th>
<th>abc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ca</td>
<td>ab</td>
<td>a</td>
<td>c</td>
</tr>
</tbody>
</table>

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?
Post Correspondence Problem

Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \}$$

A match is a sequence $i_1, i_2, \ldots, i_s$, where $t_{i_1}t_{i_2}\ldots t_{i_s} = b_{i_1}b_{i_2}\ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable.

**Pf:** by reduction from ATM

i.e. dominos simulate the sequence of configurations of ATM on $w$.
First, we'll consider MPCP where we are looking for instances that have a match that starts with the first domino.

\[ \text{MPCP} = \{ <P> \mid P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \} \} \] is a PCP that has match starting with \((t_1, b_1)\).

**Claim**: PCP is equivalent to MPCP.
Thm 5.15: PCP is undecidable.