Reductions: if we can reduce (transform) problem A into a problem B, then solving problem B gives solution to problem A.

Example: \( \text{HALT}_{TM} = \{ <M, w> \mid M \text{ is a TM that halts on } w \} \)

Thm 5.1: \( \text{HALT}_{TM} \) is undecidable.

Note: \( \text{HALT}_{TM} \) is the **halting problem**, \( \text{A}_{TM} \) is the **acceptance problem**.
Thm 5.1: $E_{TM}$ is undecidable, where

$$E_{TM} = \{ \langle M \rangle \mid M \text{ is a } TM \text{ and } L(M) = \emptyset \}$$

**Diagram:**

- **Create $M_{\text{wst}}$:**
  1. erase the tape
  2. write $w$ on the tape
  3. run $M$

- **$E_{TM}$**
  - accept if $L(M) = \emptyset$
  - reject otherwise

- **$A_{TM}$**
  - accept if $M$ accepts $w$
  - reject if $M$ rejects $w$

**Goal:** give an algo for $A_{TM}$ assuming we have a green box for $E_{TM}$

- When does $M_w$ accept its input? if $M$ accepts $w$
- What is $L(M_w) = \{ \Sigma^* \}$ if $M$ accepts $w$
  - $\emptyset$ if $M$ does not accept $w$
Thm 5.3: \( \text{REGULAR}_{TM} \) is undecidable, where

\[
\text{REGULAR}_{TM} = \{ <M> \mid M \text{ is a TM and } L(M) \text{ is regular} \}
\]
Thm 5.4: $\text{EQ}_{TM}$ is undecidable, where

$\text{EQ}_{TM} = \{ <M_1,M_2> | M_1,M_2 \text{ are TM's and } L(M_1)=L(M_2) \}$
Thm 5.4: $ALL_{CFG}$ is undecidable, where

$$ALL_{CFG} = \{ <G> \mid G \text{ is a CFG and } L(G) = \Sigma^* \}$$