Variants of Turing Machines

- what if a TM has more tapes? several heads?

This section: we’ll give detailed descriptions of our machines but not give detailed $\delta$-functions.
Multitape Turing Machines

- have to redefine $\delta$-function:

$$\delta : \mathbb{Q} \times \Gamma \times \Gamma \times \Gamma \rightarrow \mathbb{Q} \times \Gamma \times \Gamma \times \Gamma \times \{L,R,S\} \times \{L,R,S\} \times \{L,R,S\}$$

Thm 3.13: Every multitape TM has an equiv. single-tape TM.

pf idea:
- simulate multitapes on a single tape: e.g. for the above config., the single tape would contain:

state q:
$$\text{abbba} \# \text{H} \text{C} \text{d} \# \text{H} \text{a} \text{w} \text{w} \text{w} \text{w} \text{w} \text{w} \text{w} \text{w} \ldots$$

want: state p:
$$\text{bbbn} \# \text{B} \text{d} \# \text{C} \text{d} \text{w} \text{w} \text{w} \text{w} \text{w} \ldots$$

we simulate, e.g. this transition by:
- going through the tape and reading the head-pointed symbols (remember them in the state)
- go again and rewrite the symbols and move the H's
- have to redefine $\delta$-function:

$$\delta : Q \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L,R,S\})$$

Thm 3.16: Every nondeterministic TM has an equivalent deterministic TM.

Pf idea: enumerate all configurations on the tape:

- $q_0aabba \# bpaabba \# raabba \# bpaabba \#$
- $p_5babba \# bpaabba \# p_3babba$

until we find an accepting config., i.e., a config. containing the state $q_{\text{accept}}$

Notice: This is a BFS on the config. (Thus, we have to remember somewhere which config. we have already seen
- e.g., we “cross out” the config. that we already processed but they remain readable)

Question: is it OK to do a DFS?

No, see the yellow addition to the TM sketch above.
An alternative name for Turing-recognizable languages is *recursively enumerable languages*. 

An enumerator is a TM-like “printer” with no input and an extra output tape that prints all strings in a given language.
Thm 3.21: A language is Turing-recognizable iff there is an enumerator for it.