Reductions: if we can reduce (transform) problem $A$ into a problem $B$, then solving problem $B$ gives solution to problem $A$.

Example: \[ \text{HALT}_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM that halts on } w \} \]

**Thm 5.1:** \( \text{HALT}_{TM} \) is undecidable.

**Note:** \( \text{HALT}_{TM} \) is the **halting problem**, \( \text{A}_{TM} \) is the **acceptance problem**.
Thm 5.1: $E_{TM}$ is undecidable, where

$$E_{TM} = \{ <M> \mid M \text{ is a TM and } L(M) = \emptyset \}$$

Yellow: a known undecidable problem

Notice: $L(M_i) = \begin{cases} \Sigma^* & \text{if } M \text{ accepts } w \\ \emptyset & \text{if } M \text{ does not accept } w \end{cases}$
Thm 5.3: $\text{REGULAR}_{TM}$ is undecidable, where

$\text{REGULAR}_{TM} = \{ <M> \mid M \text{ is a TM and } L(M) \text{ is regular} \}$
Thm 5.4: \( EQ_{TM} \) is undecidable, where

\[
EQ_{TM} = \{ <M_1, M_2> \mid M_1, M_2 \text{ are TM's and } L(M_1) = L(M_2) \}
\]
Thm 5.4: ALL_{CFG} is undecidable, where

\[ \text{ALL}_{CFG} = \{ <G> \mid G \text{ is a CFG and } L(G) = \Sigma^* \} \]
Rice’s Thm [Problem 5.28]:

Let $p$ be a language property. If $p$ holds for some but not all languages, then the following language is undecidable:

$$R = \{ <M> \mid M \text{ is a TM and } L(M) \text{ satisfies } p \}$$