Variants of Turing Machines

- what if the tape is infinite in both directions?

1. If $L$ is recognized by the standard TM, then we can recognize it by a 2-direction infinite tape TM (simply do not use the left part of the tape).

2. If $L$ is recognized by 2DITM, then we can recognize it by a standard TM.

   - multiple possible approaches, we'll do the "fold the tape" approach.

   $\Gamma_{new} = \{ \omega \} \cup \Sigma \cup \{ \frac{0^i}{0^j} | 0^i \in \Gamma1 \text{old}, i \in \{1,2\} \} \cup \{\#\}$

   - 2 symbol

   1. Shift input right 1 position, rewrite the symbol $\Gamma \in \Sigma$ into $\frac{0^i}{0^j}$ if the first symbol is $\#$.

   2. Have states $q^T$ (for looking at the top part of the symbol) and $q^B$ (for the bottom part).

   For every original transition $\delta(q, a) = (p, x, R)$ have

   $\delta_{new}(q^T, \frac{0^i}{0^j}) = (p^T, x, R)$

   and

   $\delta_{new}(q^B, \frac{0^i}{0^j}) = (p^B, \frac{0^i}{0^j}, L)$
Variants of Turing Machines

- what if a TM has several heads?

1) if $L$ is accepted by a standard TM, then $\exists$ a dragon TM (the 2nd head simply moves right, does not rewrite the symbols)

2) if $L$ is accepted by a 2-headed dragon TM, then $\exists$ a standard TM for $L$

idea of the simulation/ construction:

1) make the 1st & 2nd tape symbol hatted ($\hat{a}, \hat{b}$) to denote the positions of the heads
2) sweep through the tape to find the hatted symbols (remember the symbols in the state), i.e. have a dedicated state for each symbol combination
3) sweep back to update the hatted symbols according to the original $\delta$-func (the dragon $\delta$)
4) repeat 2)-3) until the accept state is reached

This section: we’ll give detailed descriptions of our machines but not give detailed $\delta$-functions.
Variants of Turing Machines

- what about several tapes (and heads)?

This section: we’ll give detailed descriptions of our machines but not give detailed $\delta$-functions.
- have to redefine $\delta$-function:

Thm 3.13: Every multitape TM has an equiv. single-tape TM.
Nondeterministic Turing Machines

- have to redefine $\delta$-function:

$\delta : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L,R,s\})$

**Thm 3.16**: Every nondeterministic TM has an equivalent deterministic TM.

**idea:**
- run through the config. tree using the BFS
- if ever get to an accept. config., accept

Note: DFS would not work.

- process next config., write them on the tape,
- then mark the current config. as done and move to the next one