Suppose we have dominos of strings, e.g.:

<table>
<thead>
<tr>
<th>b</th>
<th>a</th>
<th>ca</th>
<th>abc</th>
</tr>
</thead>
<tbody>
<tr>
<td>ca</td>
<td>ab</td>
<td>a</td>
<td>c</td>
</tr>
</tbody>
</table>

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?

Is it recognizable?

YES, we can search (BFS) through the tree of possibilities, if we find one that works, we say YES (hard: to know when to say NO)
Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \}$$

A match is a sequence $i_1, i_2, \ldots, i_s$, where $t_{i_1}t_{i_2}\ldots t_{i_s} = b_{i_1}b_{i_2}\ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable. (but it is T-recognizable)
Thm: Ambiguity of CFGs is undecidable.

Example: PCP input:

Create this CFG:

\[
\begin{align*}
S & \rightarrow T | B \\
T & \rightarrow aT x_1 | aaaa5T x_1 | x_1 | aaaa6x_1 \\
B & \rightarrow aaB x_1 | aabB x_1 | aax_1 | aaax_1
\end{align*}
\]