Problem 1
Let $R = (0 \cup 11)^* \cup (10)^*$. Use the construction from the proof of Lemma 1.55 to construct an NFA $N$ such that $L(N) = L(R)$. Apply the construction literally (do not optimize the resulting NFA – keep all those $\varepsilon$ arrows in the NFA).

Note: For section 3 the construction from class differs slightly from the one given by the book (the only difference is in the Kleene’s star operation). Follow either the book’s construction or the one from class – just be consistent.

Problem 2
Follow the construction from the proof of Lemma 1.60 to generate a regular expression for the DFA $M = (\{q_0, q_1\}, \{a, b\}, \delta, q_0, \{q_1\})$ where $\delta(q_i, a) = q_{1-i}$ and $\delta(q_i, b) = q_i$, for $i \in \{0, 1\}$.

(a) Draw the initial corresponding GNFA.

(b) Draw a corresponding GNFA after removing one state.

(c) Give the final regular expression.

You may simplify the regular expressions on the transitions.

Problem 3
Let $L$ be any language over $\{a, b\}$. We define the following operation on $L$:

$$\text{ENDA}(L) = \{w \in L \mid w = ua \text{ for some string } u \in \{a, b\}^*\}.$$

Show that if $L$ is regular, then $\text{ENDA}(L)$ is also regular.

(a) Briefly discuss the idea of your construction (a short paragraph). Pictures never hurt.

(b) Formally describe your construction.

(c) Apply your construction to the language

$$L_1 = \{w \in \{a, b\}^* \mid \text{the number of } a\text{'s in } w \text{ is divisible by 3 or 4}\}.$$

For example, if your construction created an NFA for $\text{ENDA}(L_1)$ from an NFA for $L_1$, draw the two NFAs.
Problem 4

Use the pumping lemma to show that the following language is nonregular:

\[ L = \{a^i b^j c^k \mid i + j = k, i, j, k \geq 0\} \]

Provide a complete proof, follow the proof outline from the book/class.

Problem 5

Regular expressions and finite automata and their variations are used in all areas of computer science. Find a nice application, write a short summary (100-200 words), and give a link to a website with further explanation of the application.

Submit the summary and the link in a single html file in the special MyCourses dropbox. Include your name(s) and the title of your application. I will post a merged html of all the applications at the course website.