Intro to CS Theory—Homework 3
Due: Wednesday, September 28, 2011, 4pm

• You only have to explain your answer if this is stated in the question.
• Before you start on the homework, please read the rules on collaboration and submission in the syllabus.

Problem 1
Let \( N_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \delta_1, q_0, \{q_2\}) \) be an NFA with transition function \( \delta_1 \) given by the following table:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>{q_1, q_2}</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>( \emptyset )</td>
<td>{q_1, q_2}</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>{q_2}</td>
<td>{q_0}</td>
<td>( \emptyset )</td>
</tr>
</tbody>
</table>

Use the subset construction to construct an equivalent DFA \( M_2 \). Draw the state diagrams of both \( M_1 \) and \( M_2 \). You do not have to draw the unreachable states of \( M_2 \). Do not otherwise simplify \( M_2 \). Label the states with their names.

Problem 2
Let \( N_i = (Q_i, \Sigma, \delta_i, q_i, F_i), i \in \{1, 2\}, \) be two NFAs. Suppose that we want to construct an NFA \( N \) for the union of the languages \( L(N_1) \) and \( L(N_2) \). Suppose we use the following construction: take the start state of \( N_1 \) and the start state of \( N_2 \) and merge them into a single state, while keeping all transitions of both \( N_1 \) and \( N_2 \).

(a) Describe this construction mathematically, i.e., formally define the five components of \( N \) in terms of the components of \( N_1 \) and \( N_2 \). Assume that \( Q_1 \) and \( Q_2 \) are disjoint.

(b) Is this construction correct? That is, does \( N \) always accept the union of \( L(N_1) \) and \( L(N_2) \)? If yes, informally reason your answer. If not, give a concrete example of \( N_1 \) and \( N_2 \) and the resulting \( N \) (draw the state diagrams of all three NFAs) such that \( L(N) \neq L(N_1) \cup L(N_2) \).

Problem 3
Let \( M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) \) be a DFA. Construct an NFA \( N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2) \) that accepts \( L(M_1)^R \), i.e., the reverse of \( L(M_1) \).

(a) In a short paragraph informally describe how your construction works.

(b) Formally describe the 5-tuple for \( N_2 \).

(c) Apply your construction to the 8-state DFA (constructed in class) for the language of all strings over \( \{0, 1\} \) with a 1 in the third position from the end. In particular, draw the state diagram for both the DFA and the NFA obtained by applying your construction.
Problem 4

Give regular expressions for the following languages:

(a) $L_1 = \{ w \in \{0, 1\}^* \mid \text{w contains an even number of 1’s} \}$

(b) $L_2 = \{ w \in \{0, 1\}^* \mid \text{every odd position in w is a 1} \}$

(c) $L_3 = \{ w \in \{0, 1, \ldots, 9\}^* \mid \text{w is a valid decimal number, i.e., no leading zeros are allowed} \}$