Suppose we have dominos of strings, e.g.:

```
  b  a  ca  abc
  ca ab a c
```

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?

```
a  b  ca  a abc
ab ca a ab c
```

T-recognizable
(try BFS through the config tree)
Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \}$$

A match is a sequence $i_1, i_2, \ldots, i_s$, where $t_{i_1} t_{i_2} \ldots t_{i_s} = b_{i_1} b_{i_2} \ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable.
**Thm:** Ambiguity of CFGs is undecidable.

Suppose Ambiguity is decidable, let \( \text{func decideAmbig}(G) \) return TRUE if \( G \) is ambiguous, FALSE otherwise.

We will reduce PCP to Ambiguity.

**Example:**

\( \Sigma = \{a, b, *, x_1, x_2, x_3, x_4, x_5\} \)

\[
\begin{align*}
\text{top: } & S \rightarrow T \mid B \\
T & \rightarrow a T x_1 \mid a a a b T x_2 \mid a *_1 \mid a a a b x_2 \\
B & \rightarrow a a B x_1 \mid a a b B x_2 \mid a a *_1 \mid a a b *_2 \\
\text{bottom: } & S \rightarrow B \rightarrow a a B *_1 \rightarrow a a a B *_1 \rightarrow a a a a a b \ *_2 *_1 *_1,
\end{align*}
\]

\( \text{func decide PCP}(P) \)

create a CFG \( G \) from \( P \) like seen in the example.

if \( \text{decideAmbig}(G) \):

return TRUE
// PCP has a sol.
else:

return FALSE
// no sol. for the PCP instance