Variants of Turing Machines

- what if we allow the head to stay in place?

  can simulate a machine that is allowed to stay in place

  \[ \delta : (Q \times \{ \text{q}_{\text{init}}, \text{q}_{\text{acc}} \}) \times \Gamma \rightarrow Q \times \Gamma \times \{ \text{L}, \text{R}, \text{S} \} \]

  → if the LRS-machine goes L or R - do the same

  if it stays, move left/right, simulating a S

- what if the tape is infinite in both directions?

  ... w d c e c b a l b a l b a l b a l ...

  ↑

  start position for head

  many possibilities how to simulate a 2-way infinite tape by a 1-way infinite tape:

  e.g.  
  a) alternating positions: even form the right part of the 2-way infinite tape

  odd form the left part in reverse, e.g.

  \[ \text{L} \rightarrow \text{C} \rightarrow \text{A} \rightarrow \text{C} \rightarrow \text{b} \rightarrow \text{d} \rightarrow \text{b} \rightarrow \text{u} \rightarrow \text{a} \rightarrow \text{w} \rightarrow \ldots \]

  b) other ideas:

  copy left content → after right content → or always shift right when need space

  if original TM moved R on the right part, then now move 2xR left

  etc. careful when moving left of the beg. of the tape
Variants of Turing Machines

- what if a TM has several heads?
  - need to keep track of the positions of the heads if simulating by a one-headed TM:

- then the multi-headed TM goes searching for the two dotted symbols (remembering them in the state), then it goes back and changes the dotted symbols and redots the appropriate neighboring symbols, based on the 2-headed machine’s $\delta$-func.

- what about several tapes (and heads)?
  - have every odd position - first tape
    - even position - second tape
  - then dot the positions of the heads, similar to the previous two variants of TMs.

This section: we’ll give detailed descriptions of our machines but not give detailed $\delta$-functions.
Multitape Turing Machines

- have to redefine $\delta$-function:

Thm 3.13: Every multitape TM has an equiv. single-tape TM.
Nondeterministic Turing Machines

- have to redefine $\delta$-function:

$$\delta : (Q - \{q_{acc}\}) \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R\})$$

**Thm 3.16:** Every nondeterministic TM has an equivalent deterministic TM.

1. change the input into the start config for $N$
   - e.g. \[q_0, \text{waba}\]
   - start of tape symbol
2. look at the first unprocessed config on the tape:
   - based on the $\delta$-line of $N$, create subseq. configs and place them at the end of the tape
   - check first if they have been generated before
   - if yes, do not include them here
3. if accept of $N$ is ever written on the tape $\rightarrow$ accept
   - essentially a BFS of the config. tree