Topics for this week:
- backtracking: a brute-force technique that finds a solution by trying all possibilities
- related data structures
- pruning to eliminate certain possibilities from the search
- time complexity: usually really bad (exponential)
Problem: Map Coloring

How to color a map so that no two neighboring states have the same color?

Want: the smallest number of colors (to minimize costs)

Thm: Every map can be colored w. 4 colors.

We will solve the following problem: given a map and a number of colors, find if it is possible to color the map with C colors or not.
Problem: Map Coloring

Idea 1 (brute-force):

- let's try 3 colors: yellow, green, purple
- let's try all possible ways of coloring the map
  - if one of the ways is valid (no adjacent states have the same color)
    - then output that coloring

1 2 3 4 5
4 4 1 1 4  X
4 4 1 4 6  X
...
trying $3^5$ possibilities
Problem: Map Coloring

Brute-force via backtracking:

```python
def generateColors(map, currentColoring, i):
    if i > the number of states:
        check if the coloring is valid
        (check if no two adjacent states have the same color)
    else:
        # we want to color the i-th state
        for every color c:
            update current coloring so that state i is colored with color c
            generate Colors (map, current coloring, i+1)
            set the color of state i to nothing
```

Pruning:

check that neighboring states do not use color c
(if they do, do not do the recursive call)
Problem: Map Coloring

Testing:

- different maps & number of colors
  - e.g. a map with
  - 0 states
  - 1 state
  - 2-5 states
  - about 10 states
  - many states
  - number of colors: 1, 2, 3, 4, 5

Time complexity:

- if 4 colors and n states and no pruning: trying $4^n$ possibilities
  - exponential time!

- if 3 colors and 50 states (USA) and no pruning:
  - $3^{50}$ possibilities $\approx 7 \times 10^{23}$
  - if we were able to try a billion possibilities in 1 sec
    - we would need about 12 million years!
  - pruning helps a lot but it would still take days