1. Show that the following decision problem is decidable:

Given a CFG $G$, is $L(G)$ not empty?

Think of a way to mark all variables $A$ such that for some string $x \in \Sigma^*$, $A \Rightarrow^*_G x$. Explain the algorithm in detail.

2. Let $B$ be the set of all infinite sequences over $\{0, 1\}$. Show that $B$ is uncountable using a proof by diagonalization.

3. Show that the following decision problem is decidable:

Given DFAs $M_1$ and $M_2$, is $L(M_1) \subseteq L(M_2)$?

4. Consider the following problem $\text{ALL}_{\text{CFG}}$: Given a CFG $G$, is $L(G) = \Sigma^*$? This problem is undecidable. (You don’t have to prove this).

   (a) Is this problem in RE (the class of recursively enumerable languages), in coRE (the class of all languages whose complement is in RE), in both, or in neither? Explain your answer.

   (b) Use the undecidability of $\text{ALL}_{\text{CFG}}$ to show that the following problem is also undecidable:

   Given a PDA $M_1$ and an DFA $M_2$, is $L(M_1) = L(M_2)$?

5. Consider the following problem $\text{USELESS}_{\text{TM}}$: Given a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$ and a state $q \in Q$, is it the case that for any $x \in \Sigma^*$, $M$ on input $x$ does not enter $q$?

   Show that $\text{USELESS}_{\text{TM}}$ is undecidable.

6. Consider the following problem $\text{REV}_{\text{TM}}$: Given a Turing machine $M$, is it the case that $x \in L(M)$ implies $x^R \in L(M)$?

   Show that $\text{REV}_{\text{TM}}$ is undecidable.