Introduction to the Theory of Computation
Homework 2
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Read Sipser section 1.1.
Read the handout Regular Languages Part 1.

1. Sipser Exercise 1.6(a),(d),(i),(l),(m),(n).

2. Show that the set of binary integers (given as strings over \{0, 1\}) that are divisible by 3 is regular, by giving a DFA that recognizes it. Leading 0s are allowed. The empty string should be accepted. Briefly explain your answer.

3. Let \(\Sigma = \{a, b\}\), let \(k\) be a positive integer constant, and let \(L_k\) be the language defined as follows.
\[L_k = \{x \in \Sigma^* \mid \text{the number of } a \text{ s in } x \text{ is divisible by } k\}\]

   For example, \(L_2 = \{x \in \Sigma^* \mid x \text{ contains an even number of } a\}\).

   (a) Draw the transition diagram of a DFA that accepts \(L_3\).

   (b) Give a 5-tuple specifying the DFA \(M_k\) such that \(L(M_k) = L_k\).
\[M_k = (Q_k, \Sigma, \delta_k, q_0^k, F_k)\] such that ....

4. Sipser Problem 1.32.

5. Sipser Exercise 1.7(c).

6. Let \(\Sigma = \{0, 1\}\), and consider the transition table for an NFA below.

   \[
   \begin{array}{c|c|c|c}
   q & \delta(q, \varepsilon) & \delta(q, 0) & \delta(q, 1) \\
   \hline
   q_0 & \{q_3\} & \{q_4\} & \emptyset \\
   q_1 & \emptyset & \{q_0\} & \emptyset \\
   q_2 & \emptyset & \emptyset & \{q_1\} \\
   q_3 & \{q_2\} & \emptyset & \{q_6\} \\
   q_4 & \{q_0\} & \emptyset & \emptyset \\
   q_5 & \{q_3\} & \emptyset & \{q_4\} \\
   q_6 & \emptyset & \{q_5\} & \emptyset \\
   \end{array}
   \]

   Calculate \(\delta^*(q_0, 10)\).


8. Sipser Exercise 1.16. (You may use the in-class construction instead of the book construction.)