Before you start on the homework, please read the rules on collaboration and submission in the syllabus.

Read the handouts on mathematical reasoning and languages.

1. Write formal descriptions of the following sets.
   (a) The set containing the numbers 1, 10, and 100.
   (b) The set containing all integers that are greater than 5.
   (c) The set containing all natural numbers (remember the natural numbers includes zero) that are less than 5.
   (d) The set containing the string aba.
   (e) The set containing the empty string.
   (f) The set containing nothing at all.

2. Suppose $S$ is a set with $n$ elements.
   (a) How many relations are there on $S$?
   (b) How many reflexive relations are there on $S$?
   (c) How many symmetric relations are there on $S$?
   (d) How many relations are there on $S$ that are both reflexive and symmetric?

3. Sipser problem 0.12.

4. Let $\Sigma = \{a, b\}$. In each part below, a recursive definition is given of $L \subseteq \Sigma^*$. Give a simple non-recursive definition in each case. (Regular expressions as answers are not allowed.)
   (a) $L$ is the smallest set defined as follows.
      - $a \in L$
      - $xa \in L$ if $x \in L$
      - $xb \in L$ if $x \in L$
   (b) $L$ is the smallest set defined as follows.
      - $a \in L$
      - $bx \in L$ if $x \in L$
      - $xb \in L$ if $x \in L$
5. Give recursive definitions of each of the following sets.

(a) The set $\mathcal{N}$ of all natural numbers.

(b) The set $S$ of all natural numbers divisible by 7.

(c) The set $A$ of all strings in $\{a, b\}^*$ containing the substring $aa$.

6. Let $\Sigma = \{a, b\}$. Give a structurally recursive definition of function $n_b$ on $\Sigma^*$ such that for all strings $x \in \Sigma^*$, $n_b(x)$ is the number of bs in $x$.

7. Use structural induction to show that given an alphabet $\Sigma$, for any $x, y \in \Sigma^*$, $|xy| = |x| + |y|$. You may assume that string concatenation is associative. HINT: Structural induction involves only one variable; the induction should be on the second variable $y$.

8. Use structural induction to prove that given an alphabet $\Sigma$, for any $x, y \in \Sigma^*$, $(xy)^R = y^Rx^R$. You may assume that string concatenation is associative. HINT: Structural induction involves only one variable; the induction should be on the second variable $y$.

9. Prove that given an alphabet $\Sigma$, for any $\sigma \in \Sigma$, for any $y \in \Sigma^*$, $(\sigma y)^R = y^R\sigma$. HINT: Use the previous result to prove this result.

10. Use structural induction to prove that given an alphabet $\Sigma$, for any $x \in \Sigma^*$, $(x^R)^R = x$. HINT: Make use of the previous result to prove this result.