Foundations of Spiking Neural Networks

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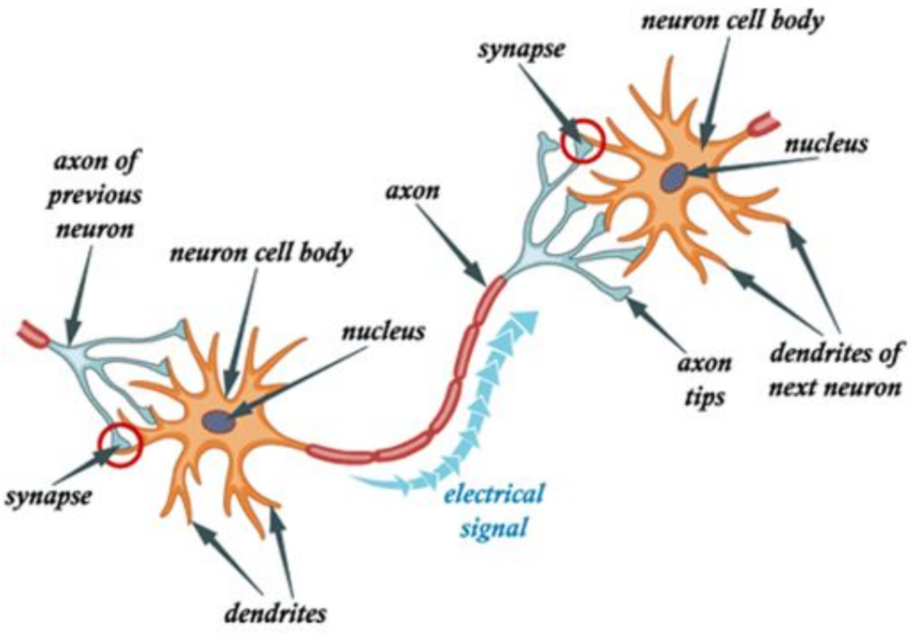
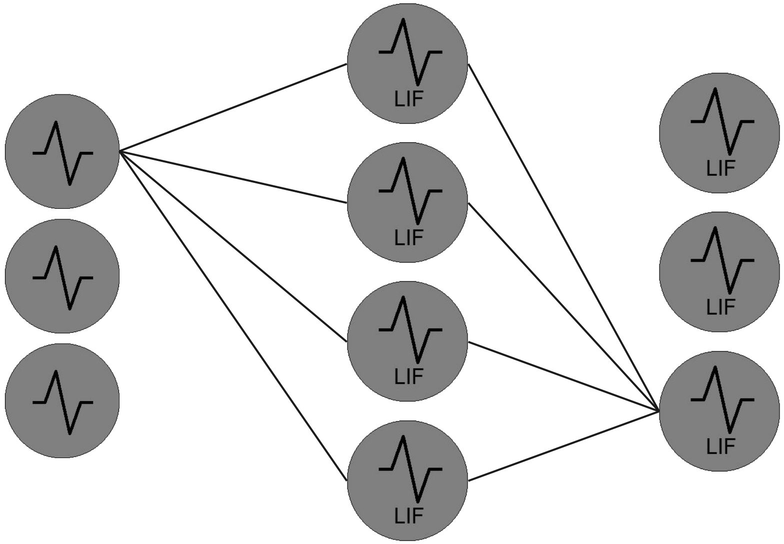
Biological Inspiration

- Neurons are the nodes of our neural networks
- Axons are the weights connecting them
- The electrical signal is not continuous through the brain



Structure

- Unlike a brain it is organized into fully connected layers
- At its most basic level many aspects of the brain are missing
- Note the input layer are not LIF neurons

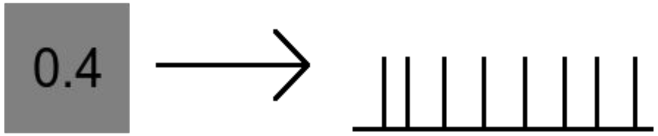


Biological Interpretation of the World

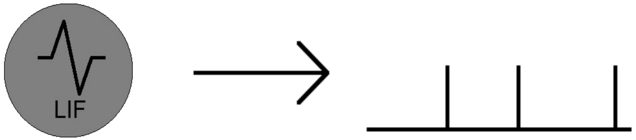
- The human eye only processes information at 60 frames per second
- Processing is not idle in the down time
- Other sensory processing occurs at different rates (order of milliseconds)

Mapping Biological Timings

- Inputs are stretched and converted to Poisson spike trains



- LIF Neurons produce spike trains instead of a singular value per image



Quick Component Recap

- Poisson spike train
  - Used to model input
- Leaky Integrate and Fire Neuron (LIF)
  - Primary neuron type
- Weights/Synapses
  - The connections between neurons

Poisson Spike Train

- Lambda (λ) is a hyper parameter
- *k* is the number of events occurring in a fixed time interval
- Note that as the time interval goes to 1 the problem has the same expected value as a Bernoulli Trial

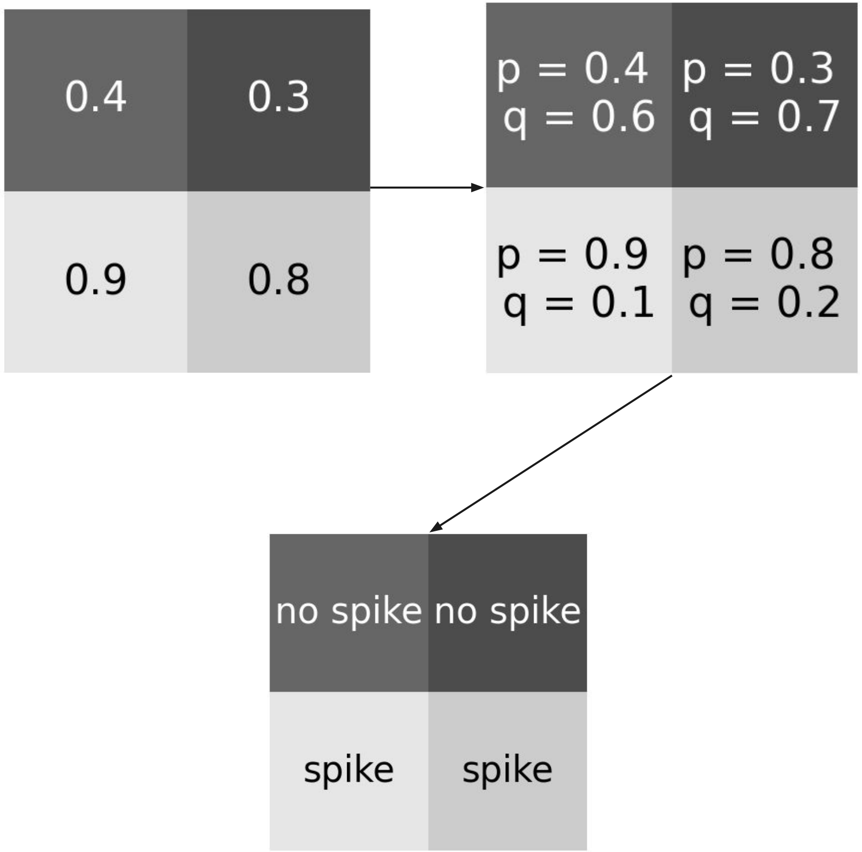
$$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{!k}$$

$$\lambda = E(X) = V(X)$$

Converting an image

- Let the normalized pixel values be equal to lambda
- Sample a single Bernoulli Trial
- Successes are treated as spikes
- Repeat for each step in the simulation

$$P_{\text{Success}} = p$$
$$P_{\text{Failure}} = q$$



Leaky Integrate and Fire Neuron

- A stateful neuron
- Slowly accumulates charge
- Fires and depolarized when threshold is passed

WARNING LOTS OF MATH AHEAD!!

LIF Current ODE

- Spikes are represented by current traveling along a wire
- Two forms, instantaneous and continuous

$$\tau_j \frac{\partial J(t)}{\partial t} = -k_j J(t_0) + (W^l s^{l-1}(t_0))$$

LIF Current ODE

$$\tau_j \frac{\partial J(t)}{\partial t} = -k_j J(t_0) + (W^l s^{l-1}(t_0))$$

$$J(t_0 + \Delta t) = J(t_0) + \frac{\Delta t}{\tau_j}(-k_j J(t_0) + (W^l s^{l-1}(t_0)))$$

LIF Voltage ODE

- Shows the change in voltage potential for a given time
- Note this looks very similar to the current equation

$$\tau_v \frac{\partial V(t)}{\partial t} = -k_v V(t_0) + rJ(t_0)$$

LIF Voltage ODE

$$\tau_v \frac{\partial V(t)}{\partial t} = -k_v V(t_0) + rJ(t_0)$$

$$V(t_0 + \Delta t) = V(t_0) + \frac{\Delta t}{\tau_v}(-k_v V(t_0) + rJ(t_0))$$

Thresholding & Depolarization

- Build a vector of spikes for a layer with a simple threshold of sigma (σ)
- Reset the voltages of every neuron that produced a spike to zero
- Leave the non-spiked neurons alone

$$s(t)_i = \begin{cases} 1 & V(t)_i \geq \sigma \\ 0 & V(t)_i < \sigma \end{cases}$$

$$V(t)_i = (1 - s(t)_i) \cdot V(t)_i$$

Adapting the Threshold

Global Threshold

- Goal is to make exactly one neuron fire per step

$$\frac{\partial \sigma}{\partial t} \propto \left(\sum_{i=1}^{N} s(t)_i\right) - 1$$

Individual Threshold

- Limits neurons that fire often to fire less often

$$\frac{\partial \sigma_i}{\partial t} \propto -\sigma_i + \beta s(t)_i$$

Neuron Dynamics

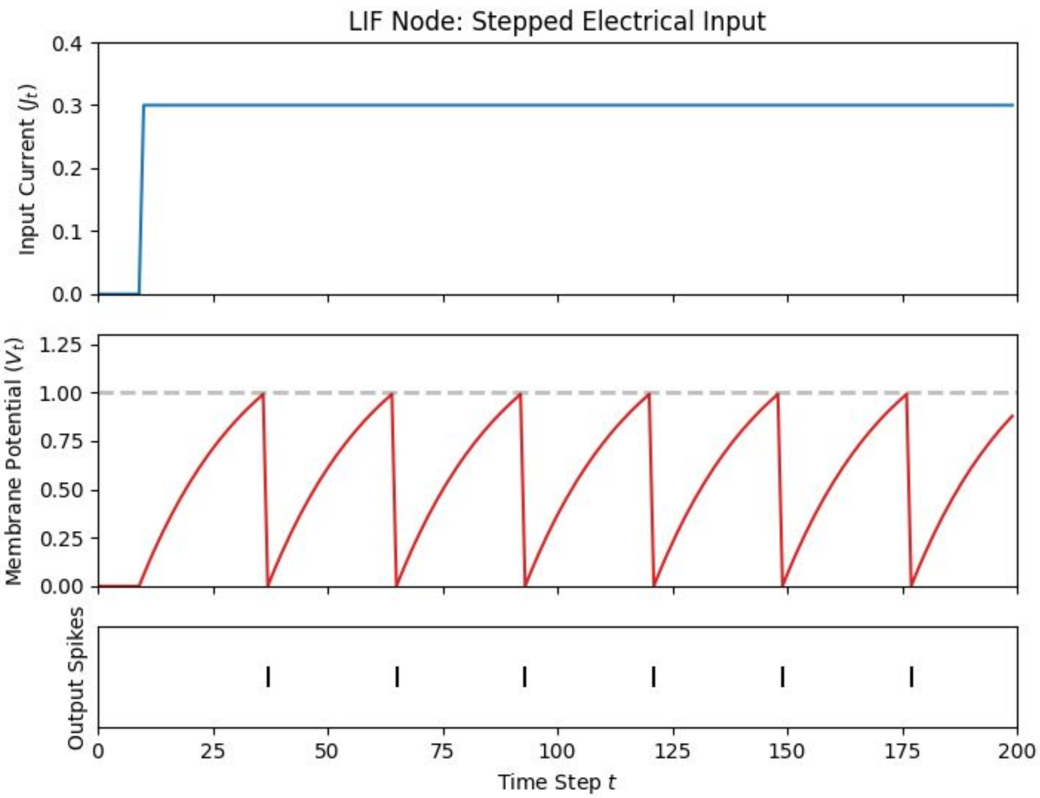


Image from NGC-Learn Walkthrough 7

Spike Dynamics

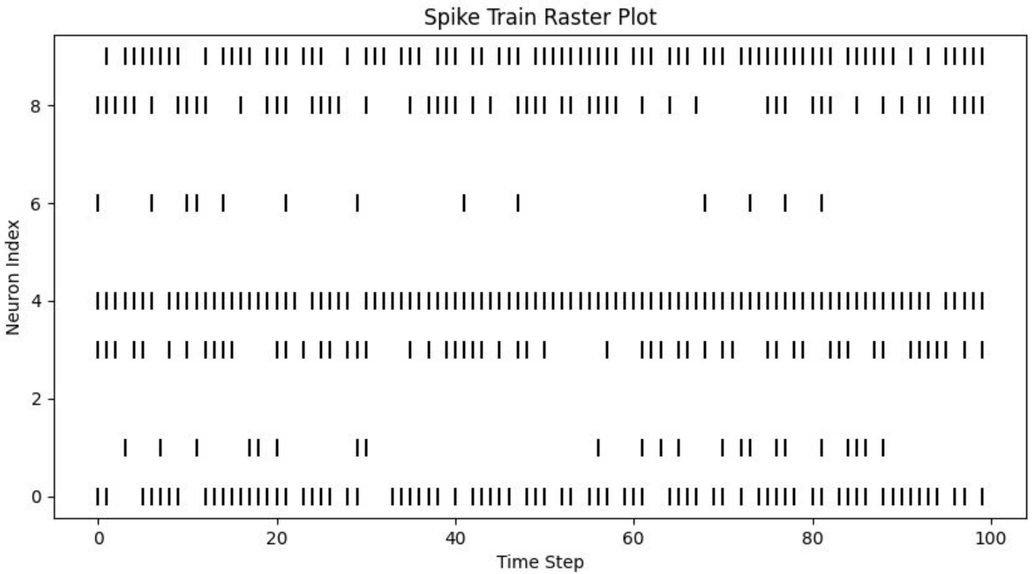


Image from NGC-Learn Walkthrough 7

$$
\begin{aligned}
\tau_j \frac{\partial J(t)}{\partial t} &= -k_j J(t_0) + (W^l s^{l-1}(t_0)) \\
\tau_j \partial J(t) &= (-k_j J(t_0) + (W^l s^{l-1}(t_0)))\partial t \\
\int_{t_0}^{t_0+\Delta t} \tau_j \partial J(t) &= \int_{t_0}^{t_0+\Delta t} (-k_j J(t_0) + (W^l s^{l-1}(t_0)))\partial t \\
\tau_j J(t)\big|_{t_0}^{t_0+\Delta t} &= (-k_j J(t_0) + (W^l s^{l-1}(t_0)))t\big|_{t_0}^{t_0+\Delta t} \\
\tau_j (J(t_0 + \Delta t) - J(t_0)) &= (t_0 + \Delta t - t_0)(-k_j J(t_0) + (W^l s^{l-1}(t_0))) \\
J(t_0 + \Delta t) - J(t_0) &= \frac{\Delta t}{\tau_j}(-k_j J(t_0) + (W^l s^{l-1}(t_0))) \\
J(t_0 + \Delta t) &= J(t_0) + \frac{\Delta t}{\tau_j}(-k_j J(t_0) + (W^l s^{l-1}(t_0)))
\end{aligned}
$$