Multi-Objective Optimization

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The Real World Contains Multiple Objectives

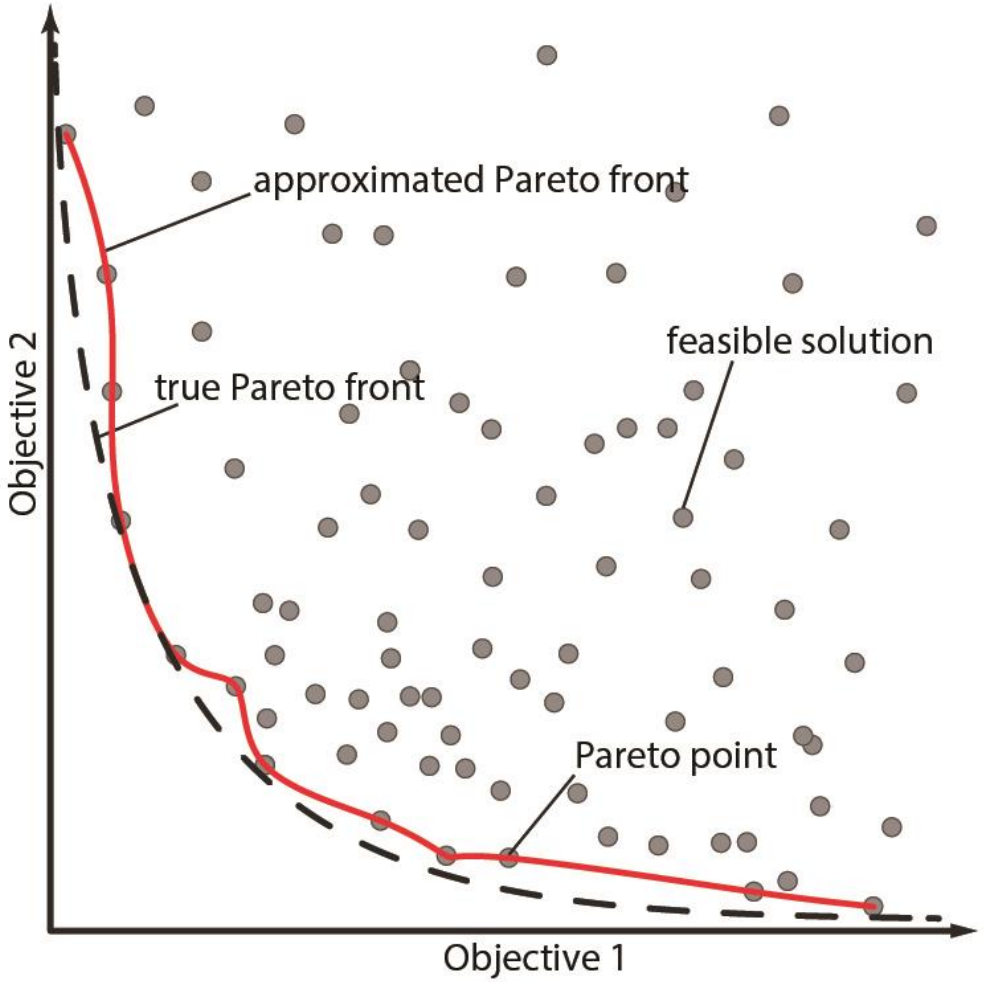
- Want to improve performance of product while minimizing cost at same time
- ***Problem***: multiple objectives may be conflicting -- trade-offs are required to reach a "balance"
  - This means multiple algorithmic runs are needed to generate a set of solutions
  - Requires good approximations of **Pareto fronts** for problem of interest

Multi-Objective Optimization

- **Pareto Optimality** = state in which system is optimized such that one objective cannot be improved without another one worsening

- Number of objectives results in more complex Pareto fronts
  - Single objective = point
  - Bi-objective = curve
  - Tri-objective = surface

- **<u>Goal</u>**: search for Pareto optimal solutions



https://iam.tugraz.at/augmented-parametrics/augp-control-system/

Multi-Objective Optimization Formulation

- **Problem**: optimize *m* objective functions, or:

$$\text{Minimize} \quad f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ \ldots,\ f_m(\boldsymbol{x})$$

  - Subject to nonlinear equality & inequality constraints:

$$h_j(\boldsymbol{x}) = 0, \quad (j = 1, 2, \ldots, J)$$
$$g_k(\boldsymbol{x}) \leq 0, \quad (k = 1, 2, \ldots, K)$$

- Can combine set of objectives into a single composite function *f*:

$$f = \sum_{i=1}^{m} w_i f_i$$

  - With:

$$\sum_{i=1}^{m} w_i = 1, \quad w_i > 0 \qquad w_i (i = 1, \ldots, m) \text{ are nonegative weights}$$

Multi-Objective (MO): Preferential Weighting

Applications of the Multi-Objective Flower Pollination Algorithm (MO-FPA)

- Spring design optimization (single objective)
- Welded beam design (single objective)
- Pressure vessel design (single obj.)
- Disc brake design (multi obj.)

- MO-FPA generally strong in approximating Pareto fronts yet…
  - …MO-FPA behavior is hard to analyze (related to central problems of FPA's dynamics as discussed earlier)

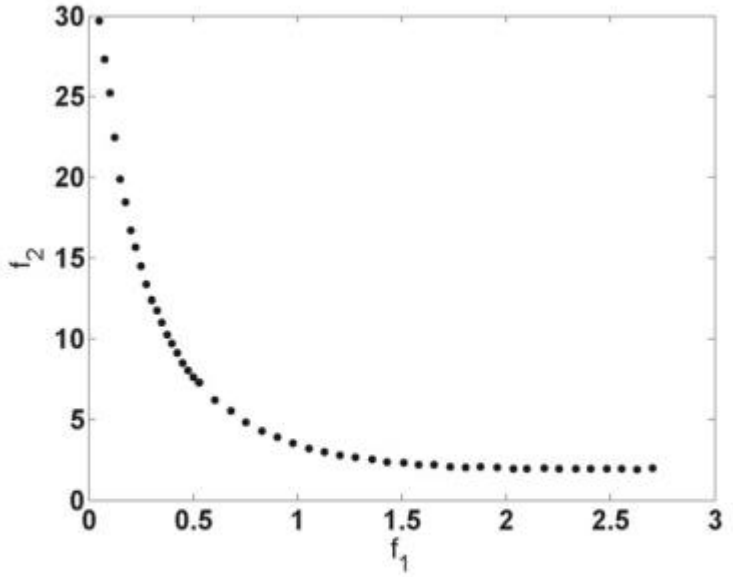


Figure 11.3 Pareto front of the disc brake design.

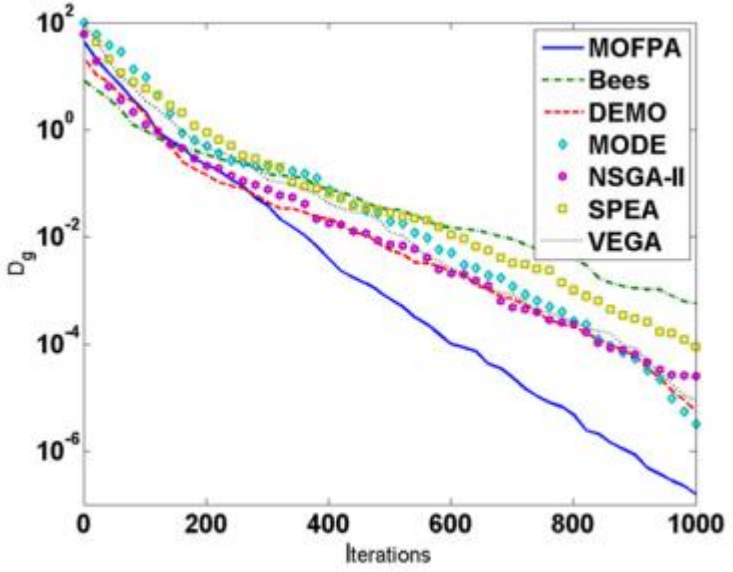
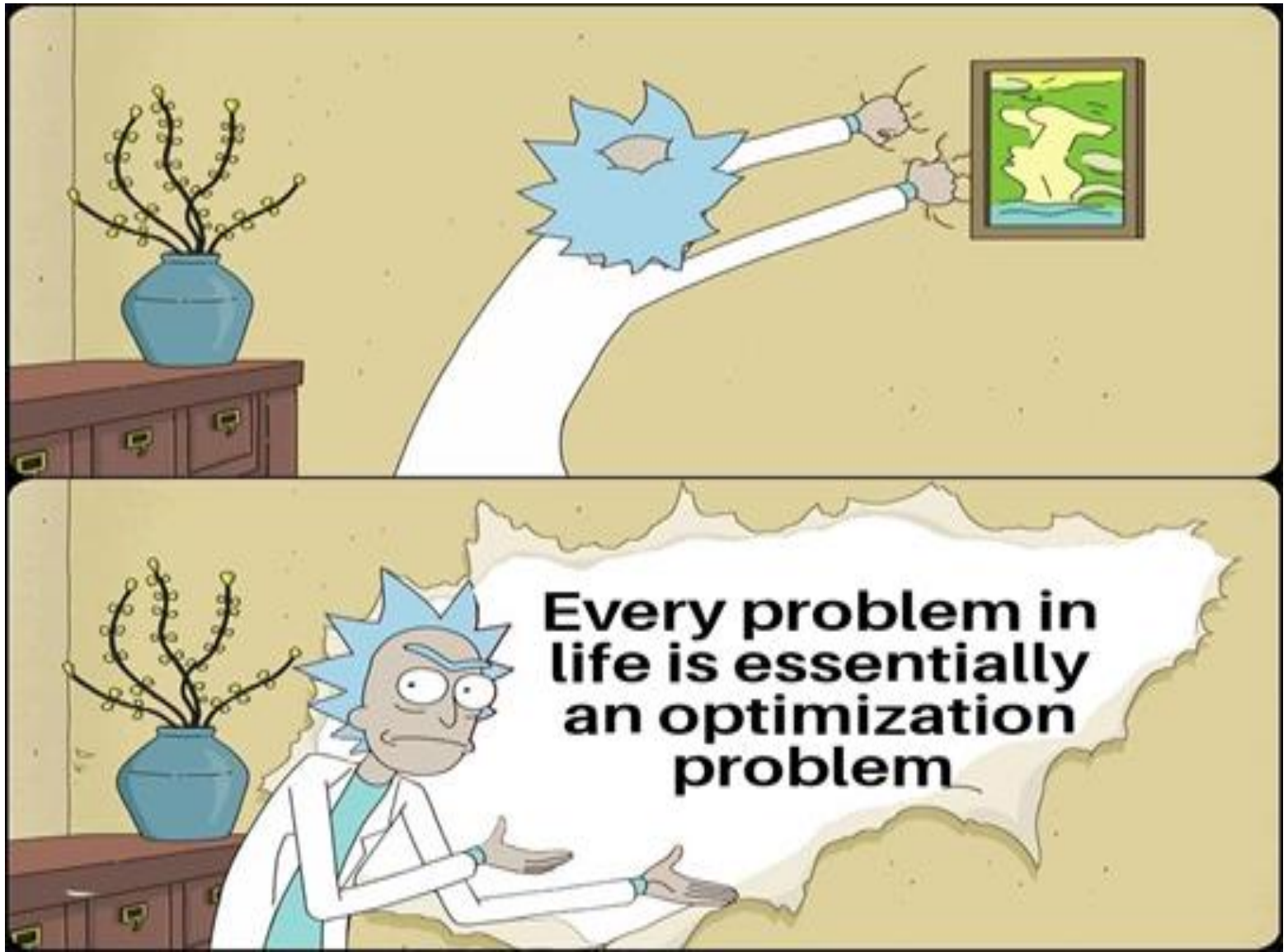


Figure 11.4 Convergence comparison for the disc brake design.

Methodology of MO Optimization

- Direct methods (very difficult to use/implement)
- Aggregation methods
  - *Weighted Sum* Method (what we sketched today)
  - The Utility Method
- Rewrite set of objectives in terms of one with others as constraints w/ imposed limits
  - The ϵ-Constraint Method
- Pareto-set approximation methods
  - NSGA-II (elitist nondominated sorting genetic algorithm)

Questions?


Every problem in life is essentially an optimization problem