On Differential Evolution

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Biologically-Inspired Intelligent Systems

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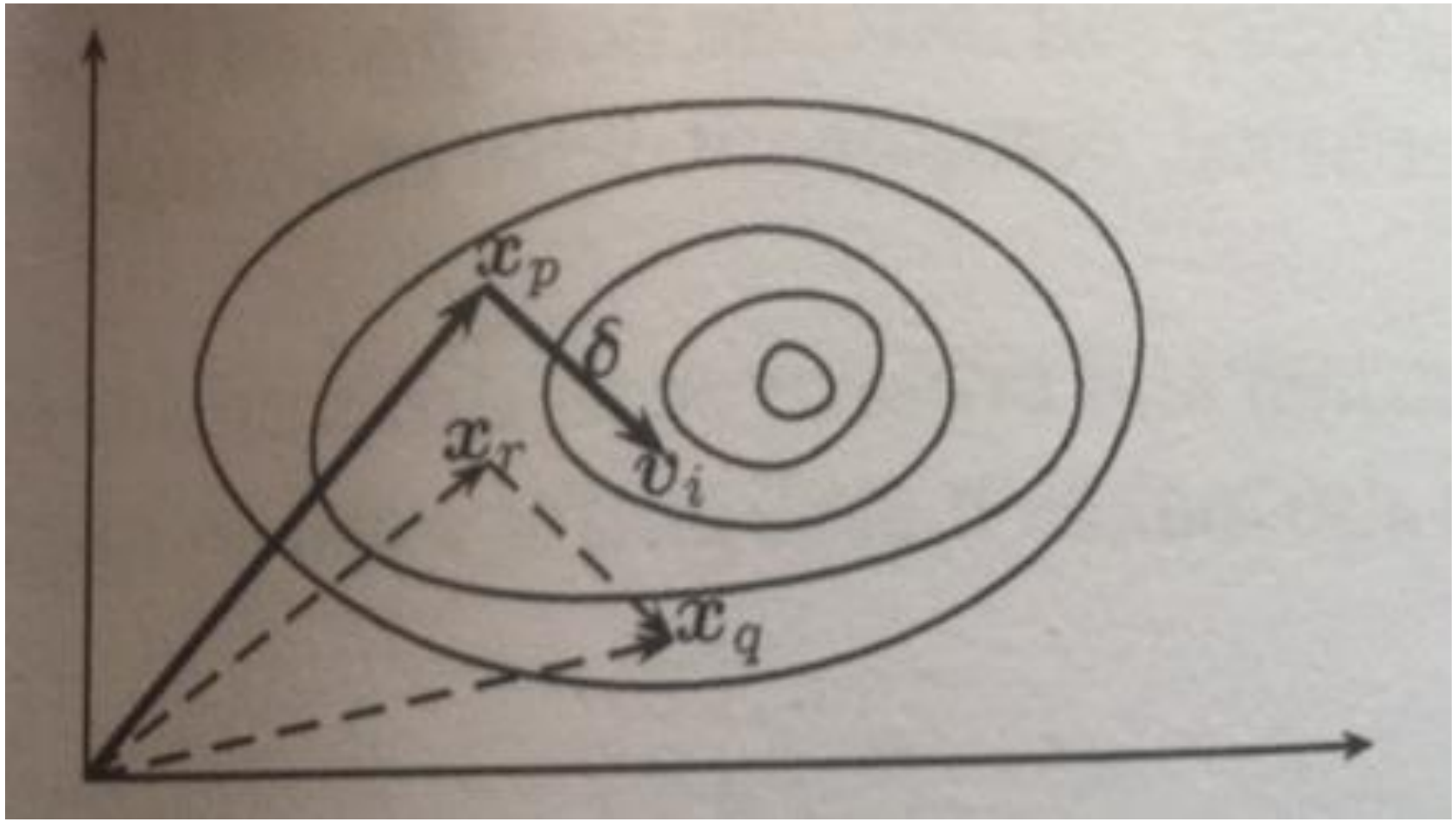
Metaheuristic: Differential Evolution (DE)

- Vector-based (population-based) algorithm, Storn & Price (1996/1997)
  - Individuals ‘evolve’ by recombination w/ other individuals & differentials between other individuals
- Devised for continuous search spaces, derivative-free
- No encoding/decoding required – real numbers are solution strings/chromosomes
- *DE/rand/1/bin*

DE General Mechanics

- Builds on the idea of genetic algorithms
- Three primary steps:
  - Mutation, crossover, selection

- Name convention: DE/x/y/z – x is mutation scheme, e.g., random (Rand) or best (Best), y is number of difference vectors, z is crossover scheme, e.g., binomial (Bin) or exponential (Exp) or either/agnostic (*)
  - **We will start with**: DE/Rand/1/*



Schematic representation the application of a perturbation/mutation in DE, according to perturbation $\boldsymbol{\delta} = \alpha(\boldsymbol{x}_q^t - \boldsymbol{x}_r^t)$ (movement along function space).

DE: A Crossover Scheme

- Crossover $C_r \in [0,1]$ controls probability/rate for crossover

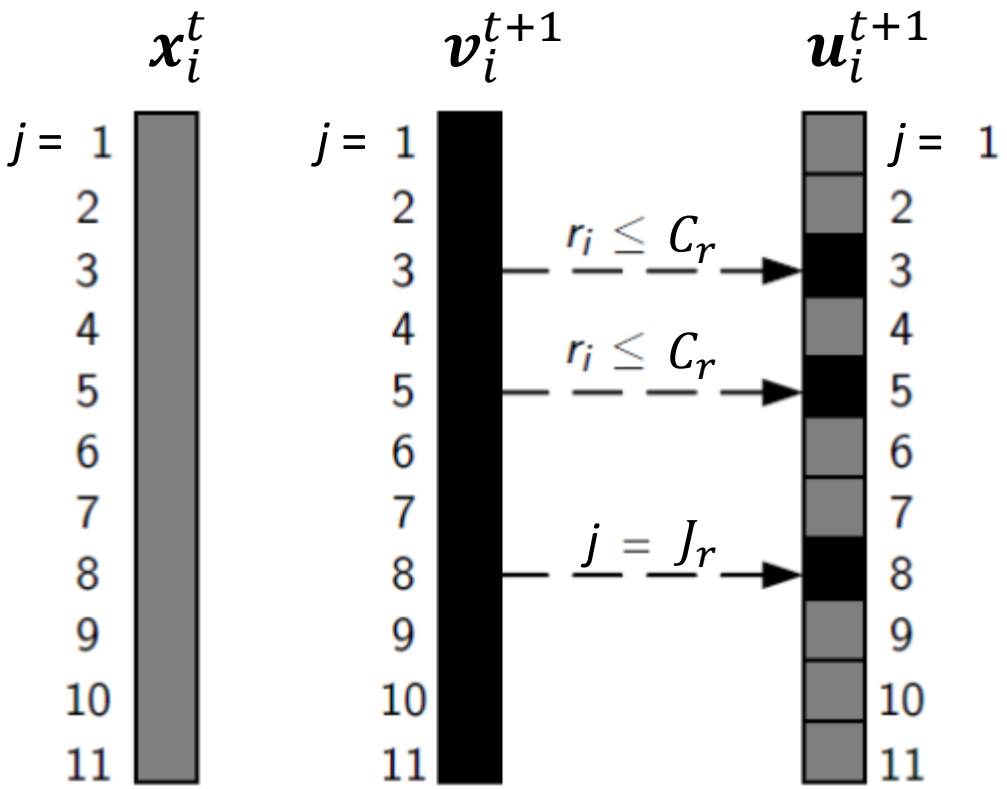
- ***Binomial*** scheme:
  - Crossover on each of *d* variables/parameters or components
  - Generate a random uniformly distributed number $r_i \in [0,1]$ and alter the *j*th dimension of $\boldsymbol{v}_i$ as follows:

$$\boldsymbol{u}_{j,i}^{t+1} = \begin{cases} \boldsymbol{v}_{j,i}, & \text{if } r_i \leq C_r, \\ \boldsymbol{x}_{j,i}^{t}, & \text{otherwise,} \end{cases} \quad j = 1, 2, \ldots, d \qquad (6.3)$$

  - Essentially, carry over dimension *j* if probability threshold not met

Binomial Crossover

- *Vector crossover visualization*



DE: Another Crossover Scheme

- ***Exponential*** scheme:
  - Segment of donor vector selected of random size *k* and random length *L* (could include multiple variables/dims)
  - Choose $k \in [0, d-1]$ and $\mathrm{L} \in [1, d]$ (uniformly), and alter dimensions/segments as follows:

$$\boldsymbol{u}_{j,i}^{t+1} = \begin{cases} \boldsymbol{v}_{j,i}, & \text{for } j = k, \ldots, k-L+1 \in [1, d], \\ \boldsymbol{x}_{j,i}^{t}, & \text{otherwise.} \end{cases} \tag{6.4}$$

- Binomial is simpler to implement, hence usually preferred

DE: Selection

- Same as in standard genetic algorithms – select the "fittest" (i.e., minimum objective value(s) of $f(\ .\ )$)
- Selection proceeds as:

$$\boldsymbol{x}_i^{t+1} = \begin{cases} \boldsymbol{u}_i^{t+1}, & \text{if } f\left(\boldsymbol{u}_i^{t+1}\right) \leq f\left(\boldsymbol{x}_i^{t}\right), \\ \boldsymbol{x}_i^{t}, & \text{otherwise.} \end{cases} \qquad (6.5)$$

- Control search efficiency by manipulating hyperparameters α and C_r

Pseudocode

Differential Evolution

Initialize the population $\boldsymbol{x}$ with randomly generated solutions
Set the weight $F \in [0, 2]$ and crossover probability $C_r \in [0, 1]$
while (stopping criterion)
  for $i = 1$ to n,
  For each $\boldsymbol{x}_i$, randomly choose 3 distinct vectors $\boldsymbol{x}_p$, $\boldsymbol{x}_r$ and $\boldsymbol{x}_r$
  Generate a new vector $\boldsymbol{v}$ by DE scheme (6.2)
  Generate a random index $J_r \in \{1, 2, ..., d\}$ by permutation
  Generate a randomly distributed number $r_i \in [0, 1]$
  for $j = 1$ to d,
  For each parameter $\boldsymbol{v}_{j,i}$ (jth component of $\boldsymbol{v}_i$), update

$$\boldsymbol{u}_{j,i}^{t+1} = \begin{cases} \boldsymbol{v}_{j,i}^{t+1} & \text{if } r_i \le C_r \text{ or } j = J_r \\ \boldsymbol{x}_{j,i}^{t} & \text{if } r_i > C_r \text{ and } j \ne J_r, \end{cases}$$

  end
  Select and update the solution by (6.5)
  end
end
Post-process and output the best solution found

Figure 6.2 Pseudo code of differential evolution.

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Improves search efficiency by ensuring at least one dimension of perturbed solution is different from the original

Meta-parameter Selection

- Extensive work shows that meta-params/hyper-params should be tuned to problem
- DE is most sensitive to scale factor α, with $\alpha \in [0.4,0.95]$ an empirically good range with a starting point of $\alpha \in [0.7,0.9]$
- $C_r \in [0.1,0.8]$ an empirically good range w/ $C_r = 0.5$ as a starting point
- For population size *n*, value should reflect dimensionality *d* of the problem, so something such as $n = 5d$ (or $10d$) → issues for high-dimensional problems
  - Can start with fixed value as starting point: $n = 40$ or 100

Some Convergence Thoughts / Issues

- Xue et al [10]* showed that λ should be large to yield better convergence
- Zaharie [13,14] – to avoid premature convergence (for any population-based algorithm), must maintain good degree of diversity [1]
  - Generally analyze/characterize the variance of DE variants (usually w/o selection) → generally lead to conclusions about meta-params such as α and how they affect variance of population solutions
- In general: when *var*(P) is decreasing/going down, DE is converging (or when *var*(P) → 0, DE has converged)
  - This convergence may be premature (i.e., not a global optima)
- ***Issue***: Population (P) diversity also depends on initial P
- ***Issue***: combinatorial problems – difficult to say if DE works well given how hard it is to discretize differential ops, etc.

* Reference/citation numbers match those in textbook ;-)

Implemented / Used In:

- **Knapsack Problem**

  - KRAUSE, Jonas; Parpinelli, R. S.; Lopes, H.S. **Proposta de um algoritmo inspirado em Evolução Diferencial aplicado ao Problema Multidimensional da Mochila**, 2012, Curitiba. Anais do Encontro Nacional de Inteligência Artificial – ENIA.

  - KRAUSE, Jonas; Cordeiro, J.A.; Lopes, H.S.. **Comparação de Métodos de Computação Evolucionária para o Problema da Mochila Multidimensional.** In: H.S. Lopes; L.C.A. Rodrigues; M.T.A. Steiner. (Org.). Meta-Heurísticas em Pesquisa Operacional. 1ed.Curitiba: Omnipax, 2013, p. 87-98.

  - KRAUSE, Jonas; Lopes, H.S. **A comparison of differential evolution algorithm with binary and continuous encoding for the MKP.** In: BRICS - Conference on Computational Intelligence, 2014, Recife. Proceedings of BRICS-CCI, 2013.

- **Scheduling Problem**

  - KRAUSE, Jonas; Sieczka, E.; Lopes, H.S. **Differential Evolution Variants and MILP for the Pipeline Network Schedule Optimization Problem.** In: LA-CCI - Congress on Computational Intelligence, 2015, Curitiba.

Differential Evolution (Naming)

Many Variations:

- best/n/bin
- best/n/exp
- random/n/bin
- random/n/exp
- current/n/bin
- current/n/exp

Parent Selection / Number of Pairs / Recombination

- In general:
  - Perform binary or exponential recombination between the current individual and another individual modified by a scaled difference between *n* pairs of other individuals

Questions?