Stochasticity and Simulated Annealing

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Stochasticity and Random Walking

The Central Limit Theorem

- In many cases, for i.i.d. random variables, sampling distribution of standardized sample mean tends towards a standard normal distribution even if original variables are not normally distributed

Lindeberg–Lévy CLT — Suppose $\{X_1, \ldots, X_n\}$ is a sequence of i.i.d. random variables w/ $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$. Then, as n approaches infinity, the random variables $\sqrt{n}(\bar{X}_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$:

$$\sqrt{n}\left(\bar{X}_n - \mu\right) \xrightarrow{a} \mathcal{N}\left(0, \sigma^2\right).$$

Towards Simulated Annealing (SA)

- Hill-climbing that never makes downhill (or “bad”) moves, gets stuck in local maxima easily (incomplete)
- Pure random walks (moving to successor state uniformly at random from set of successors) is complete BUT extremely inefficient
- Why not combine both? You get SA (“shaking” algo)
- We introduce temperature T that decreases over time
    - At start, we allow for move bad moves when T is high
    - Towards end, we permit few(er) bad moves when T is low
- VSLI layout problems (historical), factory layouts, etc.

Simulated Annealing

Questions?