On Linear Regression

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Introduction to Machine Learning

CSCI-335

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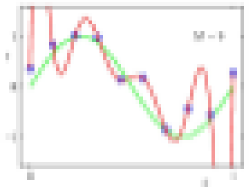
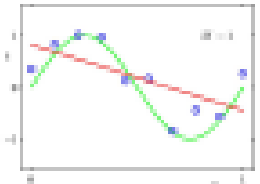
Special Thanks: Sargur N. Srihari, Andrew Ng, Carlos Guestrin

Goal: Minimize Expected Loss

- We have decomposed expected loss into sum of (squared) bias, a variance and a constant noise term
- There is a trade-off between bias and variance
  - Very flexible models have low bias and high variance
  - Rigid models have high bias and low variance
  - Optimal model has has the best balance





Machine Learning Algorithms

	Supervised Learning	**Unsupervised Learning**
Discrete	Classification	Clustering
Continuous	Regression	Dimensionality reduction

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Recap: Nearest Neighbor Classifier

- **Training data**

$$\left(x^{(1)}, y^{(1)}\right), \left(x^{(2)}, y^{(2)}\right), \cdots, \left(x^{(N)}, y^{(N)}\right)$$

- **Learning**

Do nothing.

- **Testing**

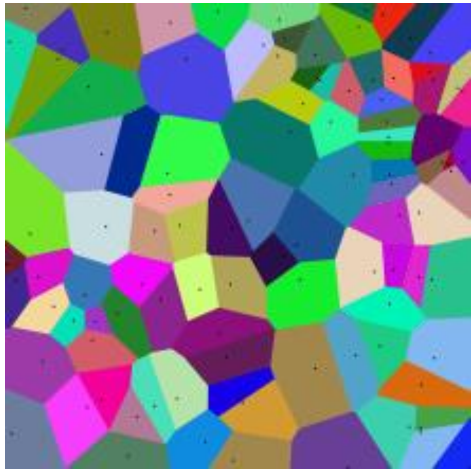
$$h(x) = y^{(k)}, \text{ where } k = \operatorname{argmin}_i D\left(x, x^{(i)}\right)$$

Recap: Instance/Memory-based Learning

1. *A distance metric*
   - Continuous? Discrete? PDF? Gene data? Learn the metric?
2. *How many nearby neighbors to look at?*
   - *1? 3? 5? 15?*
3. *A weighting function (optional)*
   - Closer neighbors matter more
4. *How to fit with the local points?*
   - Kernel regression (not discussed, but good to be aware of)

Slide credit: Carlos Guestrin

Things to Remember



- Supervised Learning
  - Training/testing data; classification/regression; hypothesis
- K-NN
  - Simplest learning algorithm
  - With sufficient data, very hard to beat “strawman” approach
- Problems with K-NN
  - Curse of dimensionality
  - Not robust to irrelevant features
  - Slow NN search: must remember (very large) dataset for prediction

Linear Regression

- **Model representation**
- Cost function
- Gradient descent
- Multivariate regression

<u>**Regression**</u>

real-valued output

Training set

↓

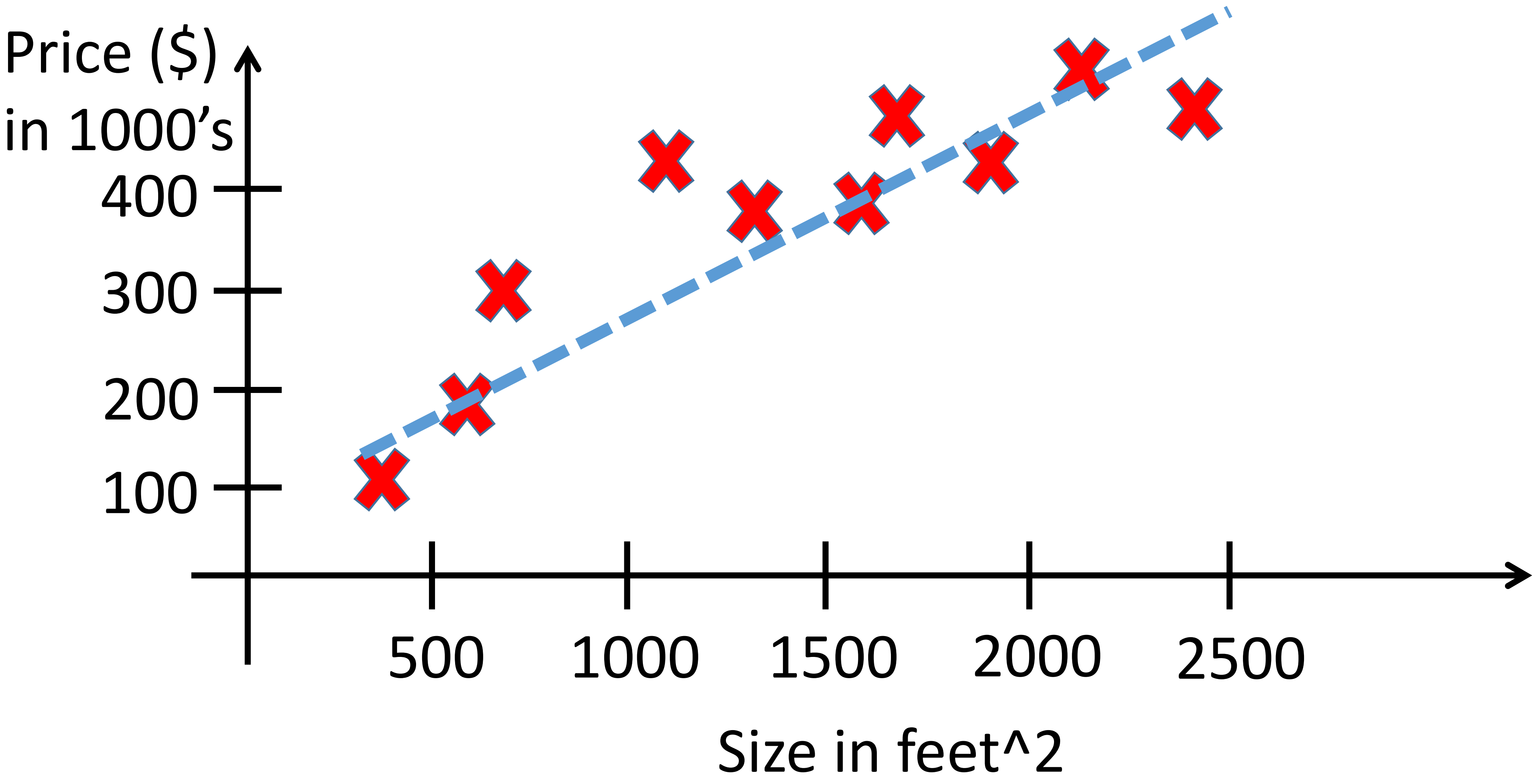
Learning Algorithm

↓

x → h → y

Size of house — Hypothesis — Estimated price

House Pricing Prediction



Training Set

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

$m = 47$

- Notation:
  - m = Number of training examples
  - x = Input variable / features
  - y = Output variable / target variable
  - (x, y) = One training example
  - $(x^{(i)}, y^{(i)}) = i^{th}$ training example

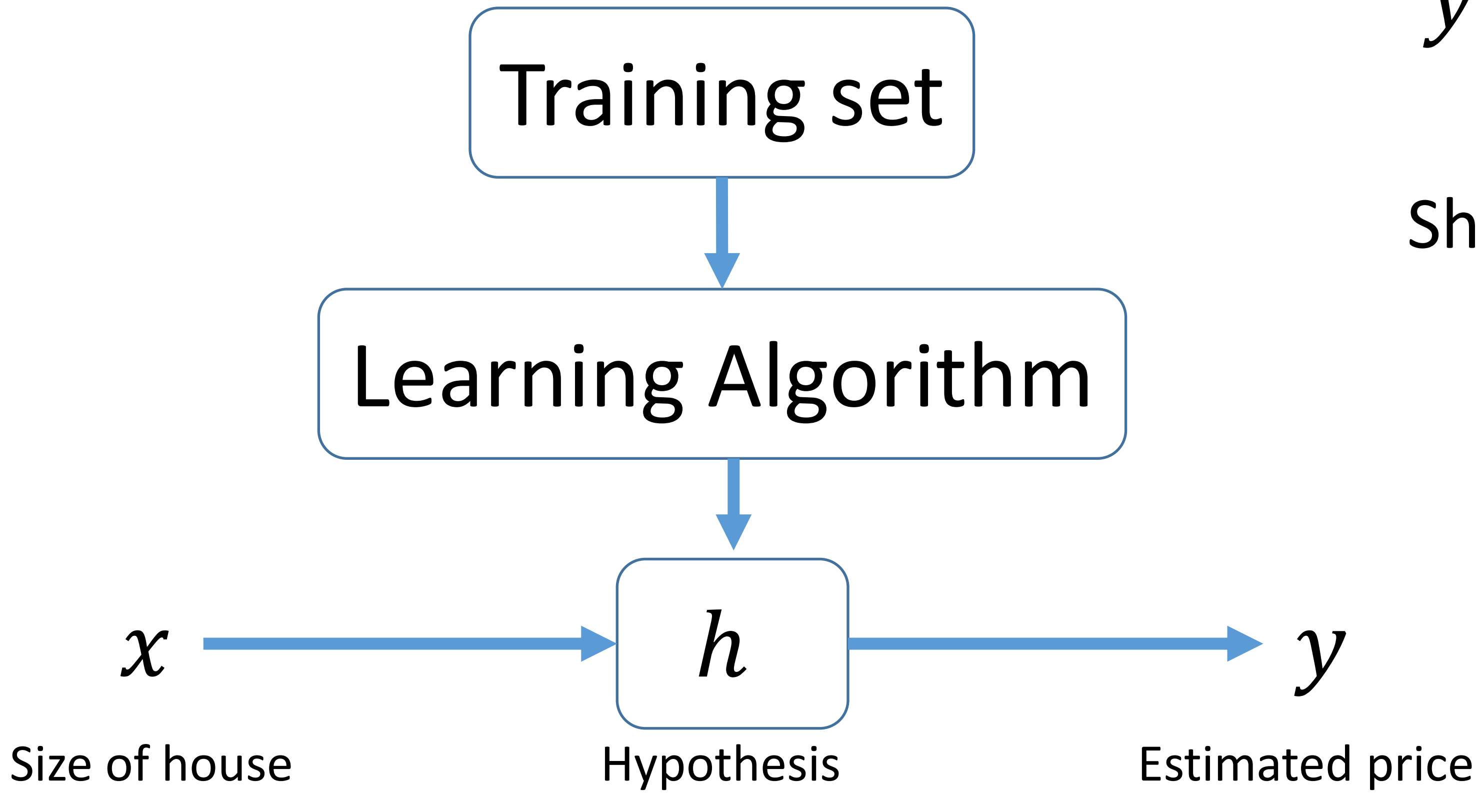
Examples:

$x^{(1)} = 2104$

$x^{(2)} = 1416$

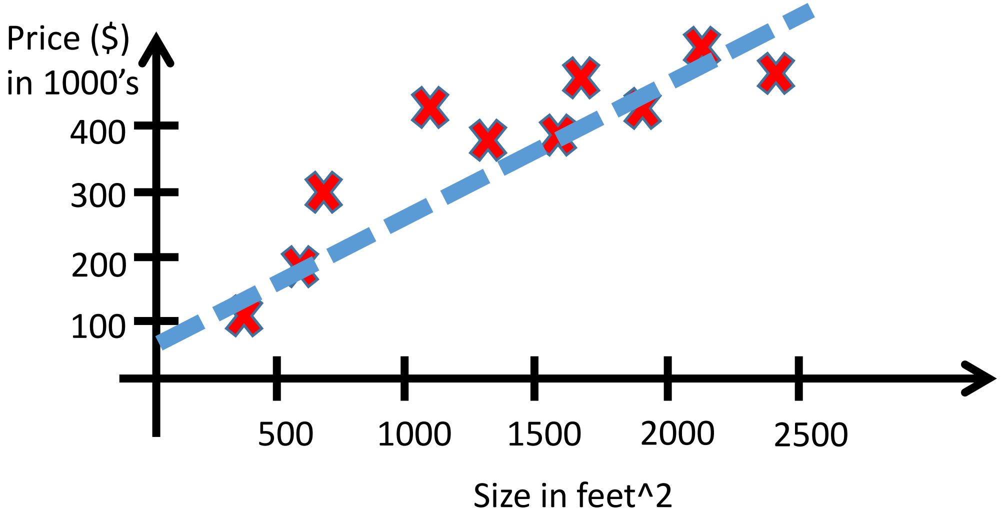
$y^{(1)} = 460$

Model Representation



$$y = h_\theta(x) = \theta_0 + \theta_1 x$$

Shorthand $h(x)$



Univariate linear regression

Linear Regression

- Model representation
- **Cost function**
- Gradient descent
- Multivariate regression

Training Set

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
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…	…

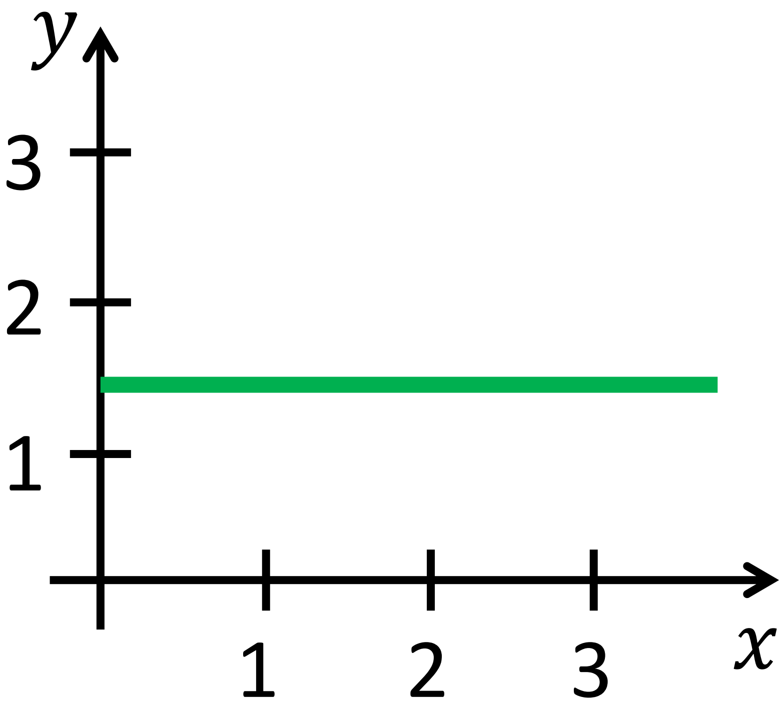
m = 47

- Hypothesis $h_\theta(x) = \theta_0 + \theta_1 x$

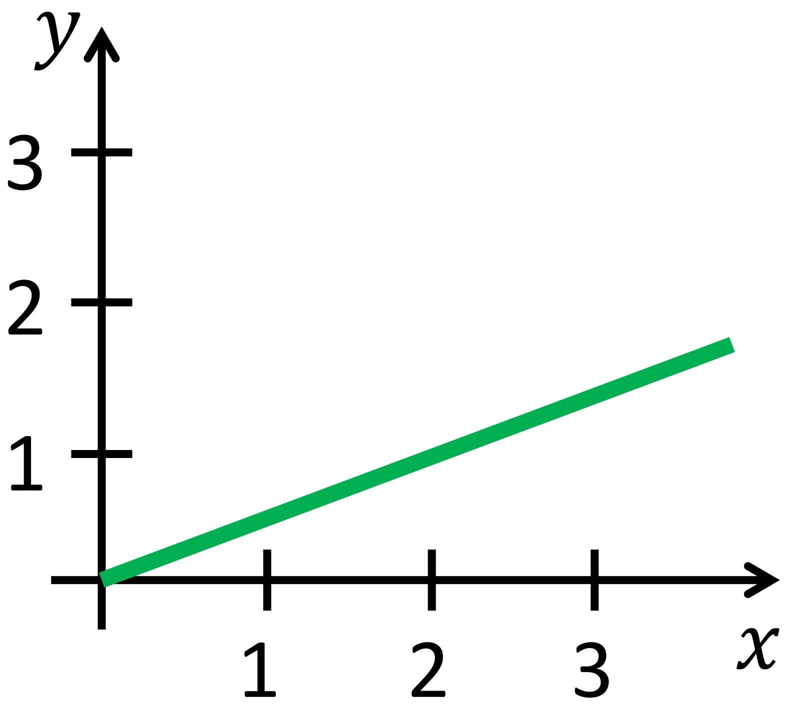
θ_0, θ_1: parameters/weights

How to choose θ_i's?

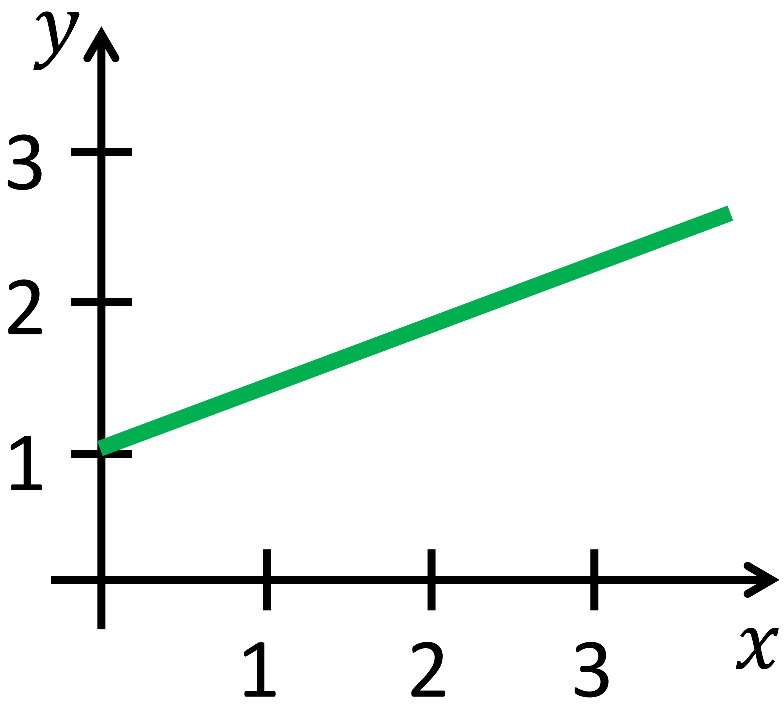
$$h_\theta(x) = \theta_0 + \theta_1 x$$



$\theta_0 = 1.5$
$\theta_1 = 0$



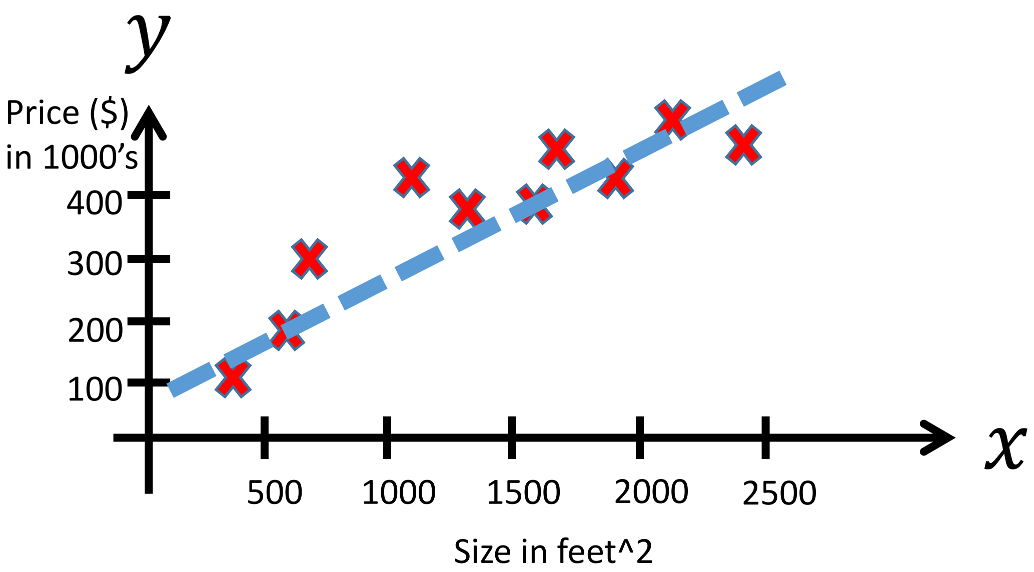
$\theta_0 = 0$
$\theta_1 = 0.5$



$\theta_0 = 1$
$\theta_1 = 0.5$

Cost Function

- Idea:
  Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training example (x, y)



$$\underset{\theta_0,\theta_1}{\text{minimize}} \; \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

$$h_\theta\left(x^{(i)}\right) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

$$\underset{\theta_0,\theta_1}{\text{minimize}} \; J(\theta_0, \theta_1)$$

Cost function

Simplified

- **Hypothesis:**

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- **Hypothesis:**

$$h_\theta(x) = \theta_1 x \qquad \theta_0 = 0$$

- **Parameters:**

$$\theta_0, \theta_1$$

- **Parameters:**

$$\theta_1$$

- **Cost function:**

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

- **Cost function:**

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

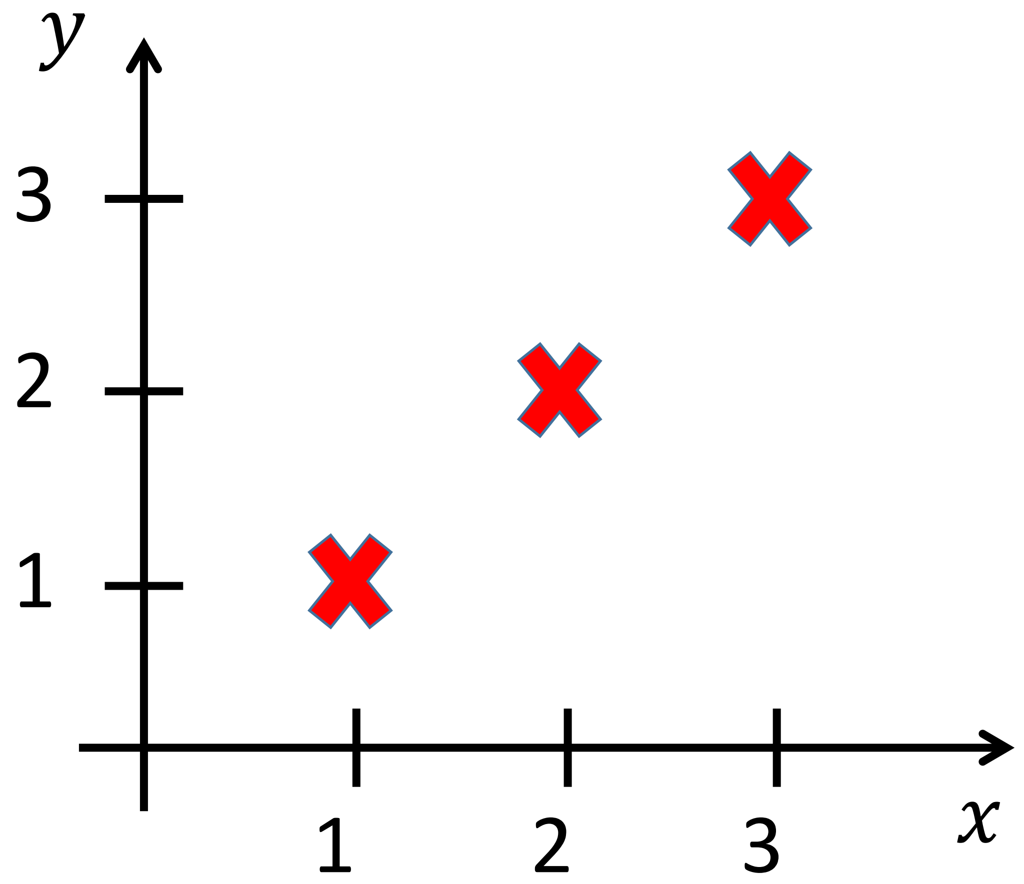
- **Goal:**

$$\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$$

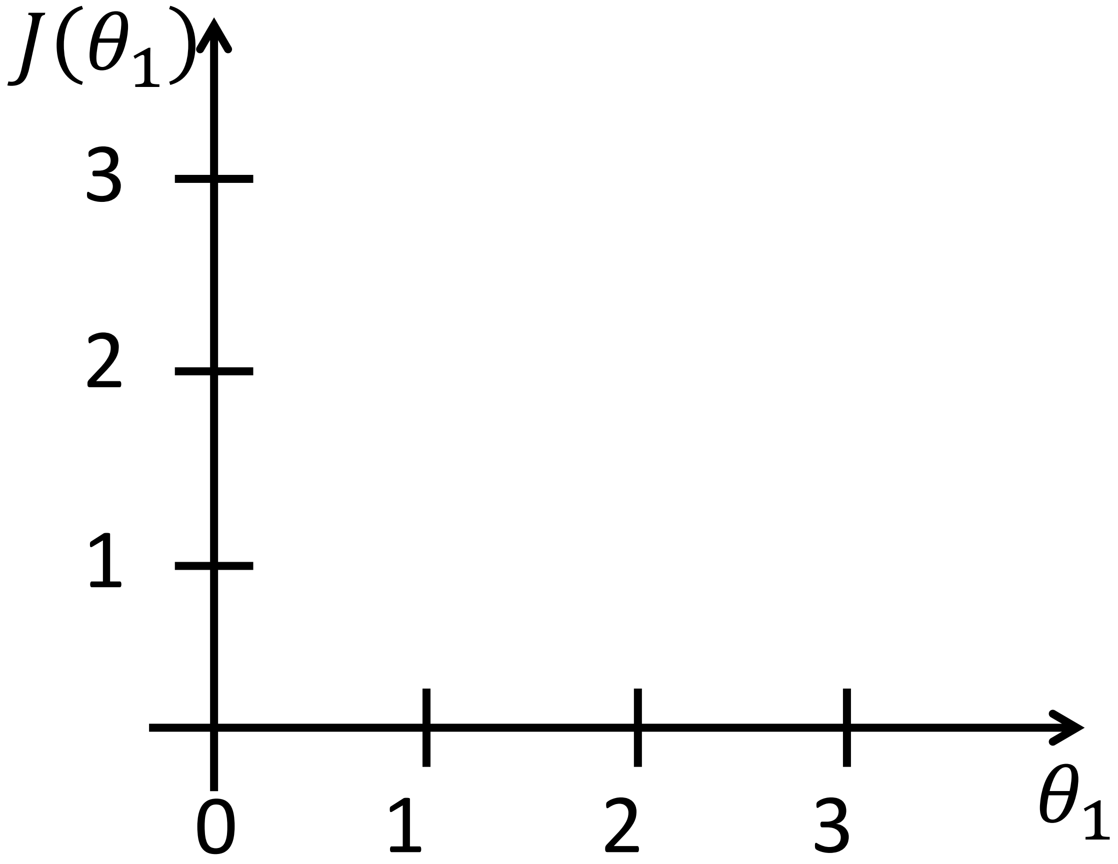
- **Goal:**

$$\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_1)$$

$h_\theta(x)$, function of x



$J(\theta_1)$, function of θ_1



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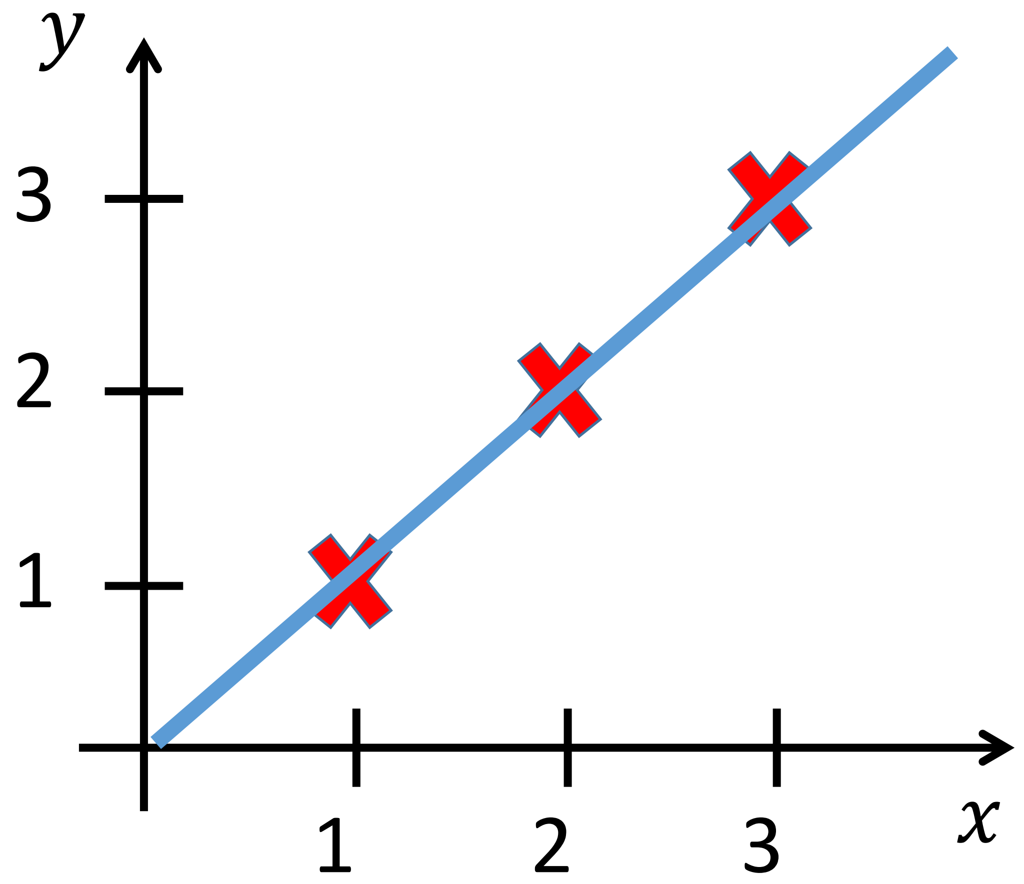

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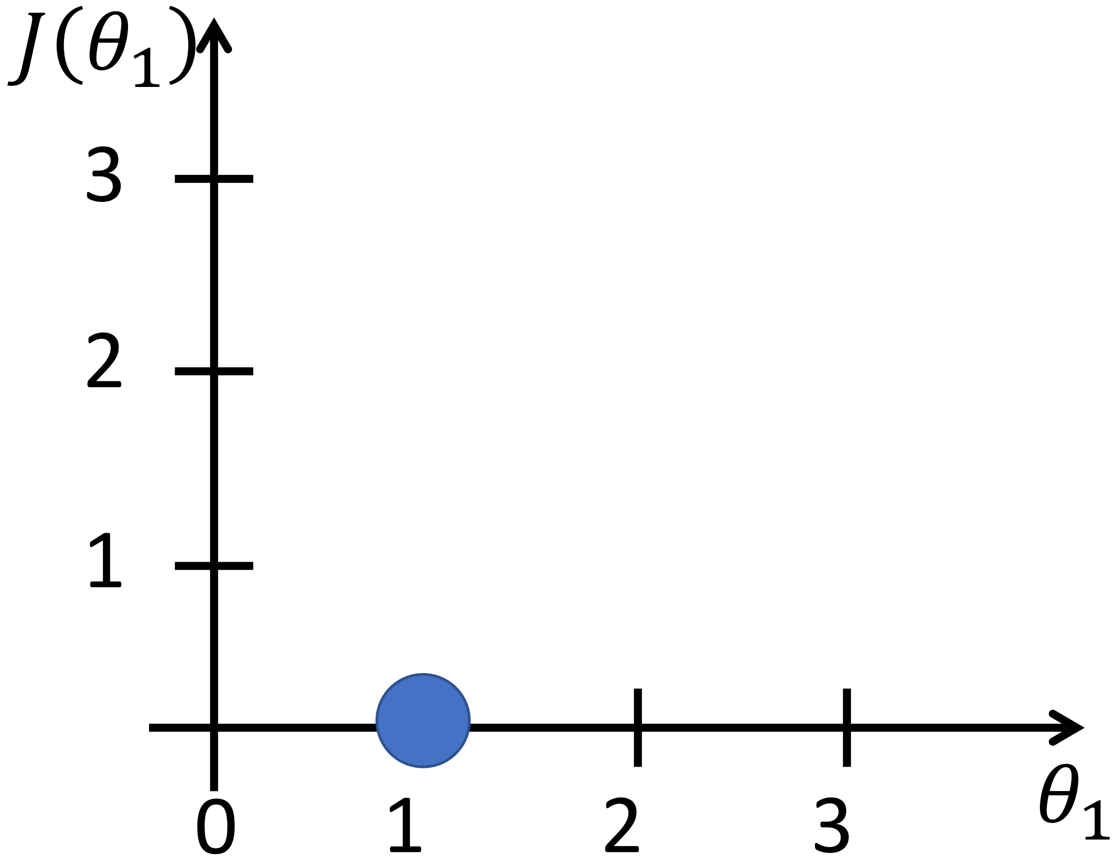

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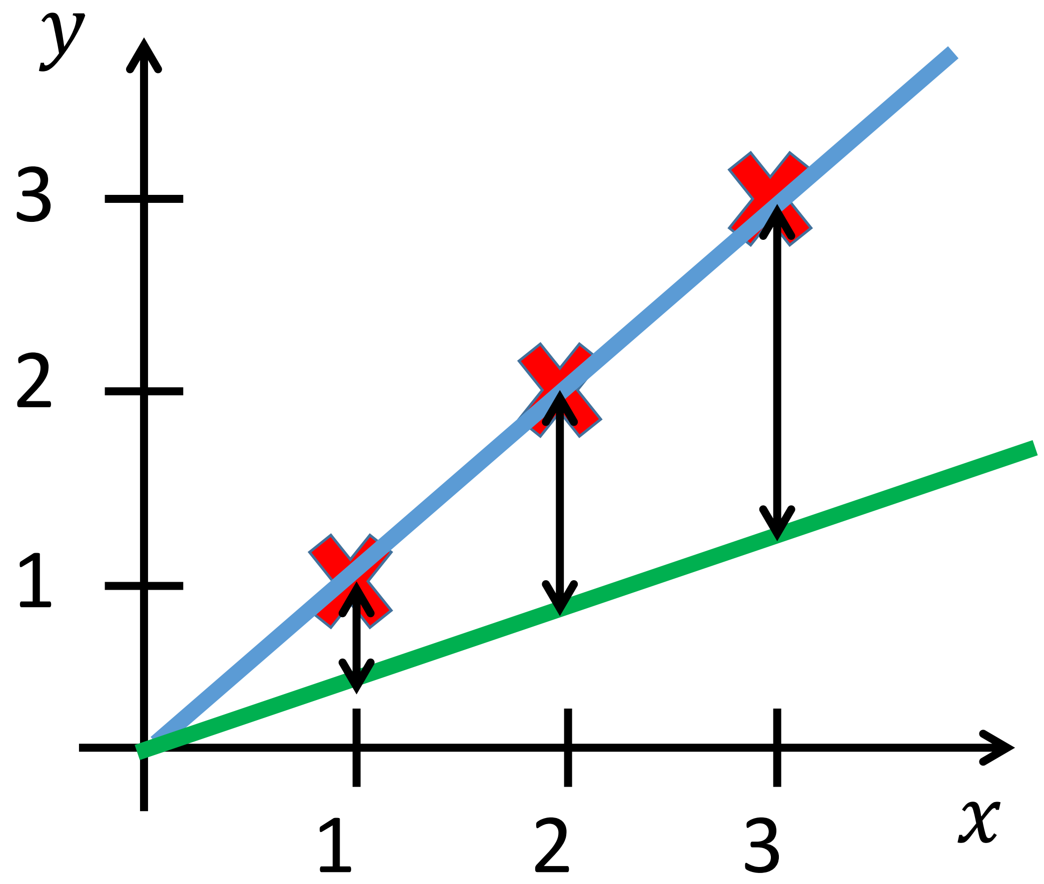

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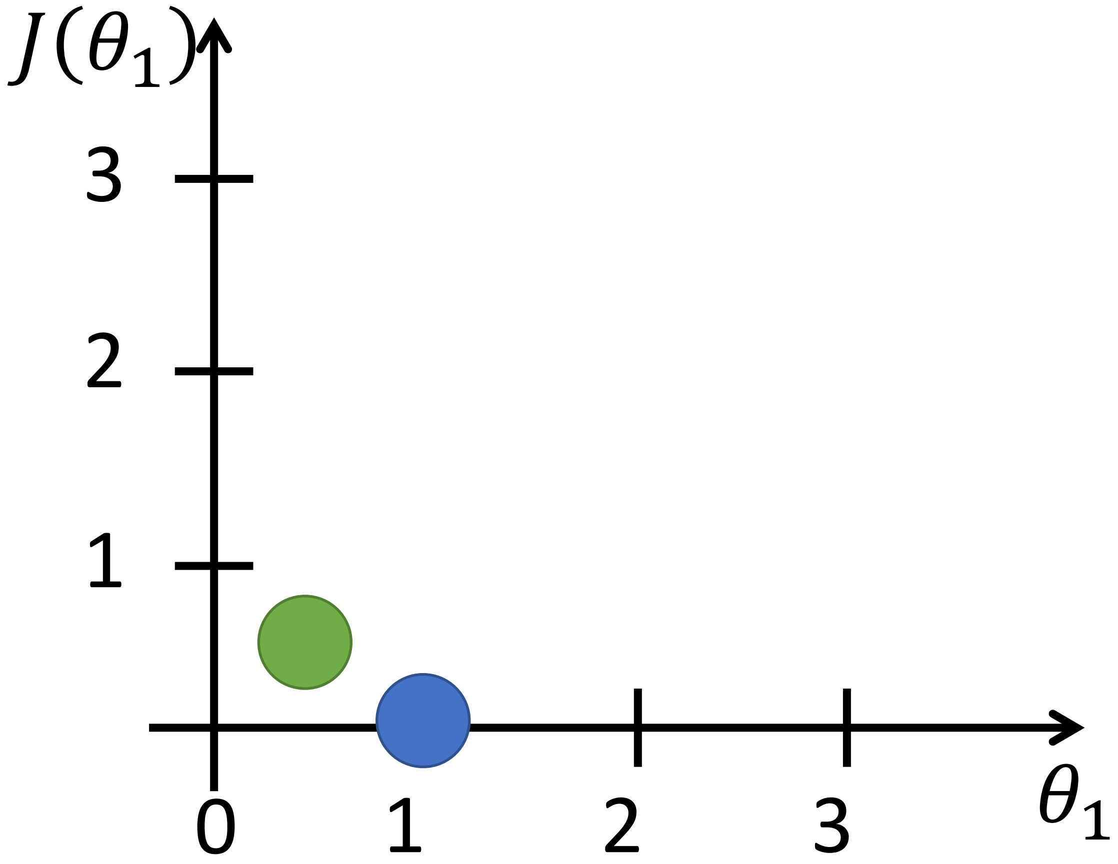

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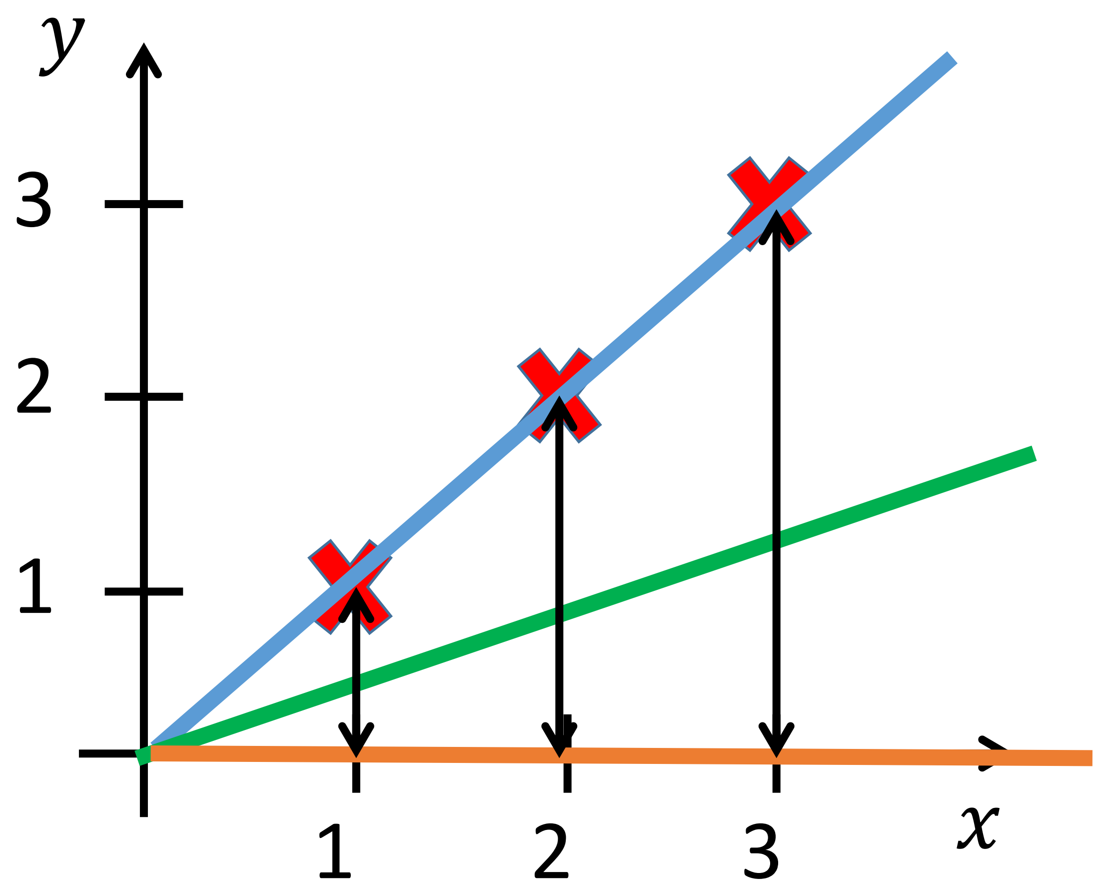
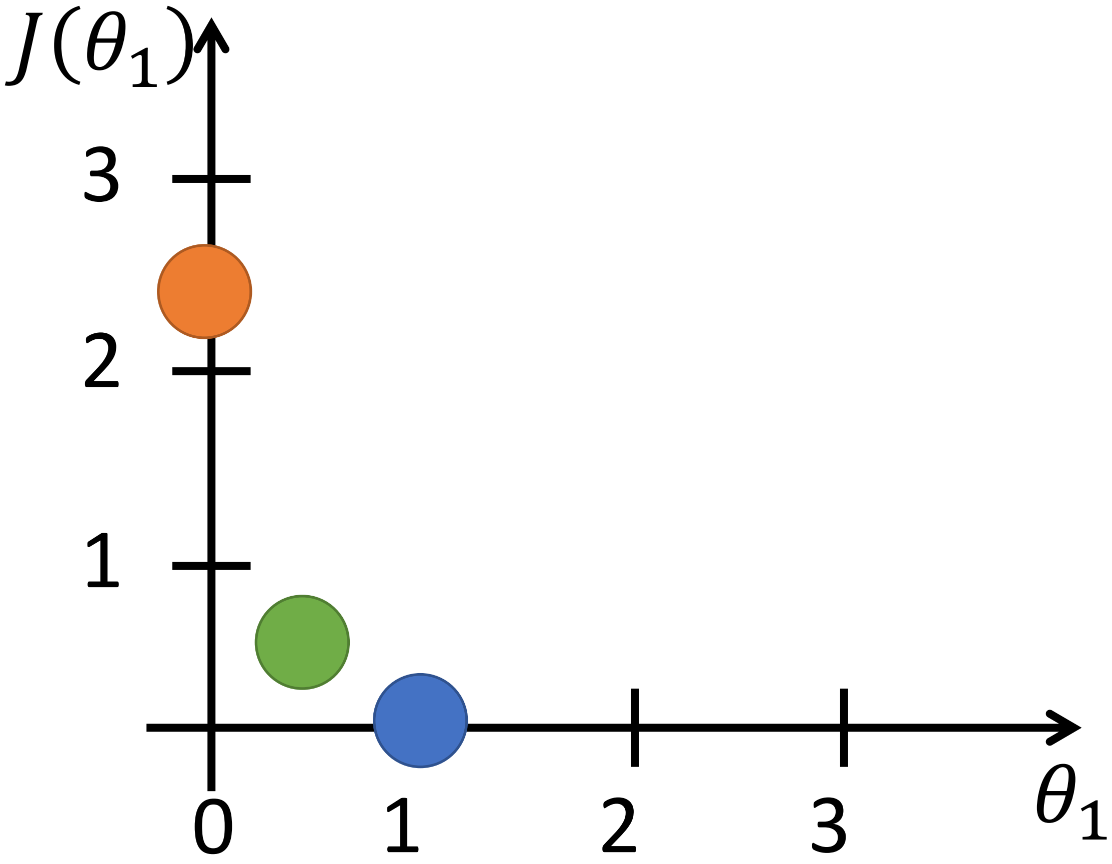
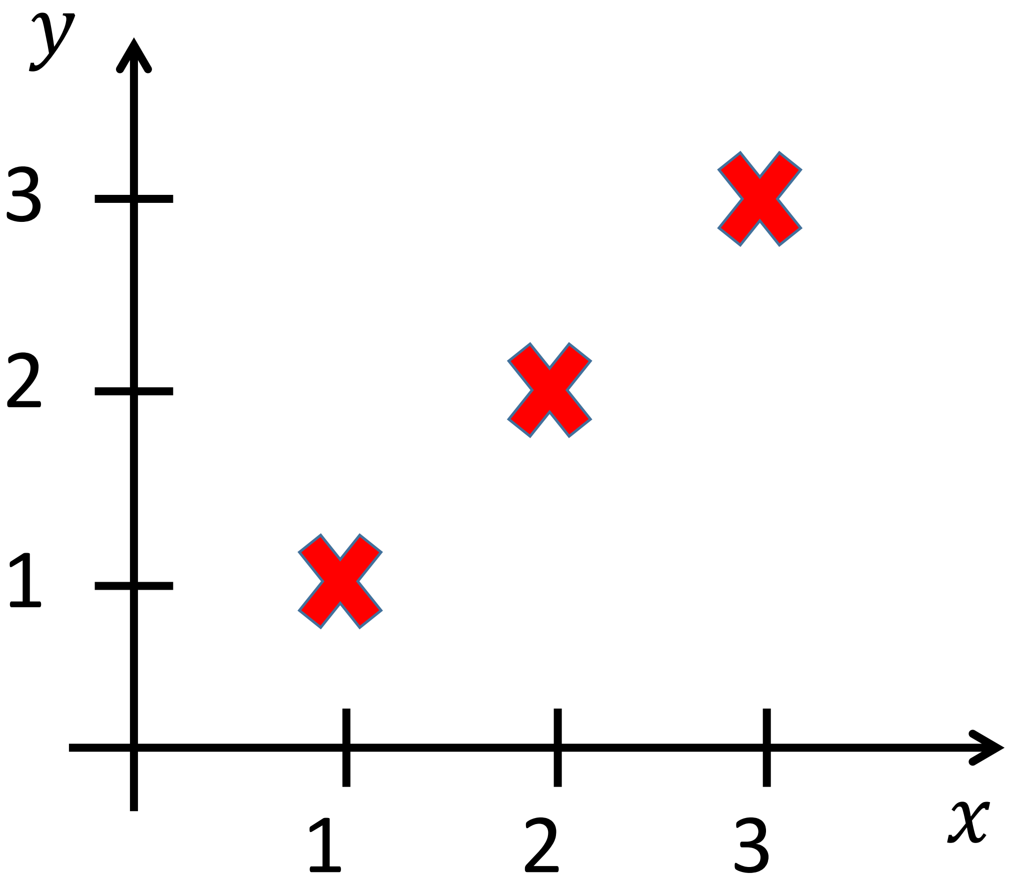
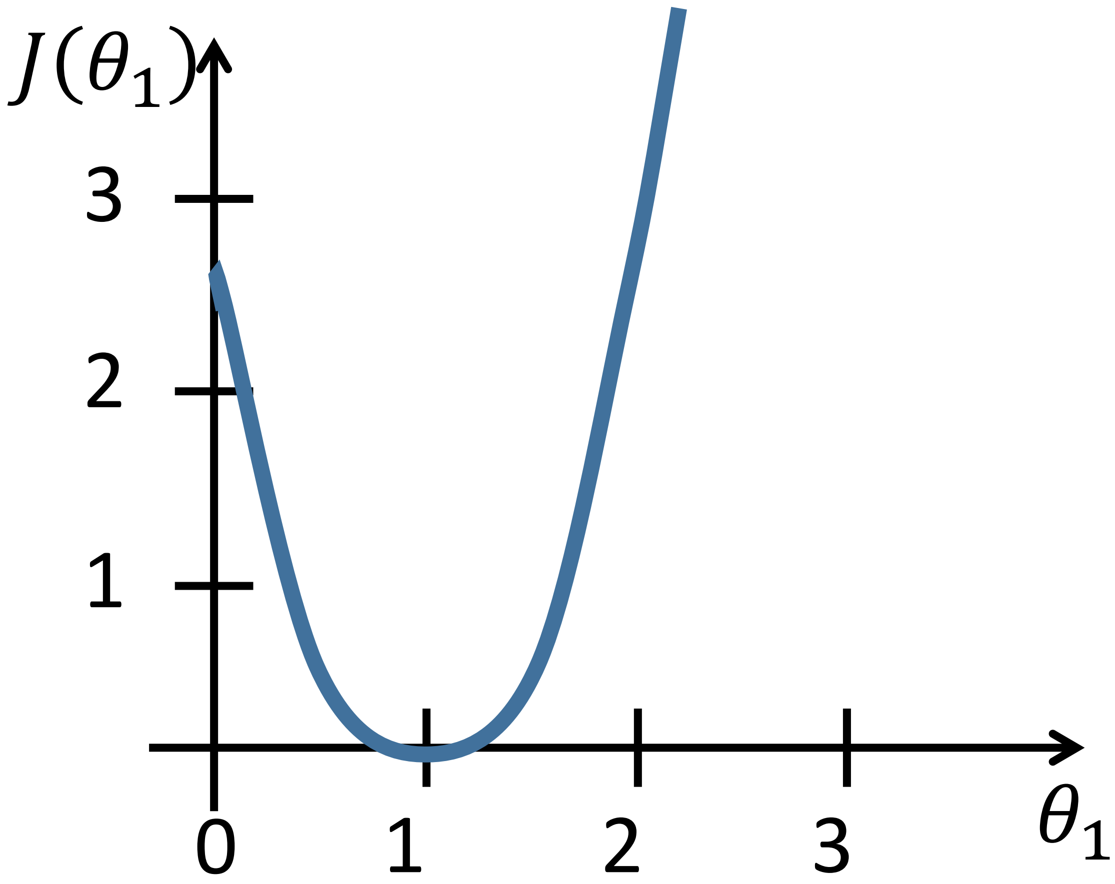


$J(\theta_1)$, function of θ_1



$h_\theta(x)$, function of x | $J(\theta_1)$, function of θ_1





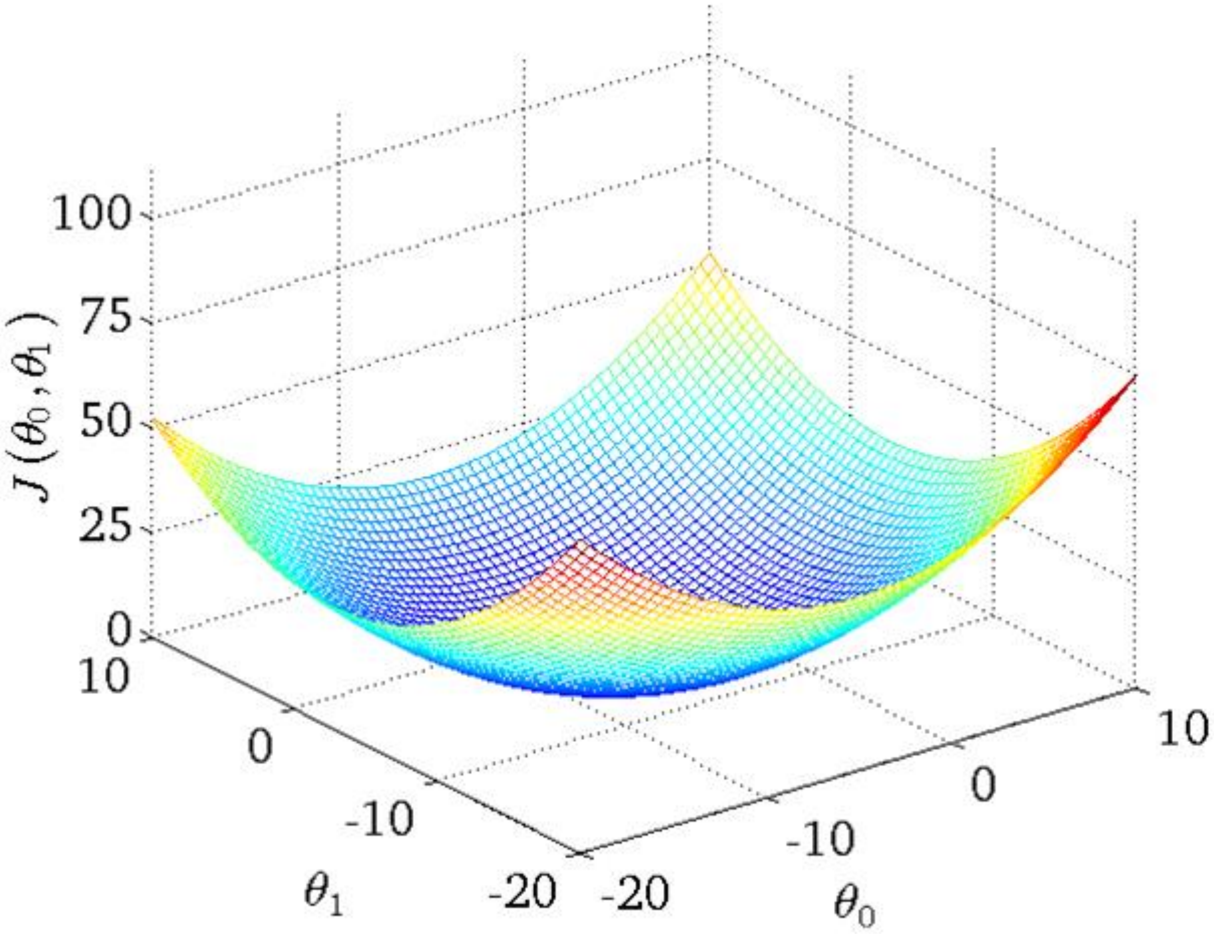
- **Hypothesis:** $h_\theta(x) = \theta_0 + \theta_1 x$

- **Parameters:** θ_0, θ_1

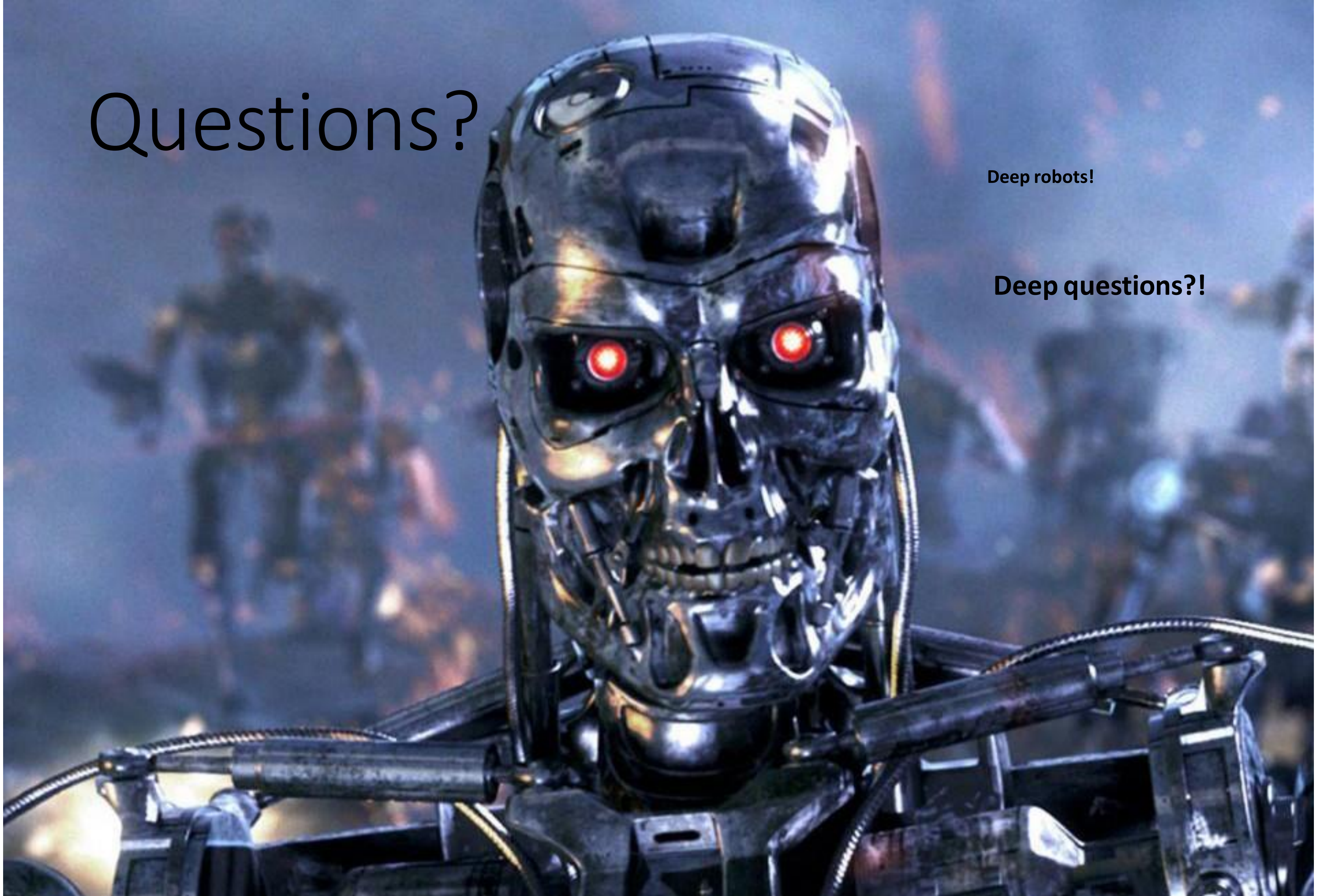
- **Cost function:** $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$

- **Goal:** $\underset{\theta_0, \theta_1}{\text{minimize}} \; J(\theta_0, \theta_1)$

Cost Function Surface



Questions?



Deep robots!

Deep questions?!