Logarithms

In spherical trigonometry

- there are many formulas that require multiplying two sines together, e.g., for a right spherical triangle
- sin b = sin B sin c
- In the 1590's it was known (as the method of prosthaphaeresis) that by using the trigonometric identity this could be simplified to
  - sin b = sin B sin c = [cos(B - c) - cos(B + c)] / 2
- replacing a multiplication by an addition

This probably encouraged people to look for methods to replace multiplication with simpler operations

Also it was known that to multiply two numbers of the series

- 1, 2, 4, 8, 16, ... or 2^0, 2^1, 2^2, 2^3, 2^4, ...

we just need to add the exponents of the two numbers

Napier

Napier (1550-1617) using geometrical reasoning published in 1614 a 56 page text and 90 pages of tables describing logarathms

- His original tables were of logarathms of sines
- The base of his logarithms were somewhat 1/e although there was a factor of 10^7 and the log of 1 was not 0

Logarithms to the base e = 2.718281828459045... are known as Napierian logarithms or natural logarithms

Briggs

Henry Briggs obtained a copy of Napier's publication and became enthusiastic about logarithms

- He met and had several discussions with Napier and improved them by
  - Making the log of 1 to be 0
  - Making 10 be the base of the logarithm (although integers were preferred in the tables so logs were multiplied by 10^{10})

Logarathms to the base 10 are also known as common logarithms or Briggsian logarithms

Basic operations with logarithms

The antilogarithm of n is the number whose logarithm is n

To multiply two numbers we add their logarithms (and take the antilogarithm)

- log ab = log a + log b

To divide two numbers we subtract their logarithms (and take the antilogarithm)

- log a/b = log a - log b

To take a square root we divide the logarithm by 2 (and take the antilogarithm)

- log $\sqrt{a}$ = ½ log a

To take an nth root we divide the logarithm by n (and take the antilogarithm)

- log $\sqrt[n]{a}$ = (1/n) log a

To raise to the nth power we multiply the logarithm by n (and take the antilogarithm)

- log a^n = n log a

Using a table of logarithms

Modern tables of (base 10) logarithms are arranged to give logarithms of numbers in the range of 1 to 10

- To get the logarithm of a number outside of this range we multiply by a power of ten (shift the decimal point) until the number is in the range of the table
- We then add the number of places we shifted to the table value effectively multiplying by the proper power of 10
  - Remember that the logarithm of 10 is 1

If the table does not have exactly the number we are taking the logarithm we must interpolate between the neighboring values

- For example, if we want the log of 2.3456 but the table only has the log of 2.345 and 2.346 we
  - take the difference between the log of 2.345 and the log of 2.346
  - divide this difference by 10
  - multiply by the 6
  - and add the result to the log of 2.345

Many tables have a list of "proportional parts" to help in this

- log 2.345 = .37014
- log 2.346 = .37033
- The difference is 19
- The "6" entry in the proportional part table of 19 is 11
- log of 2.3456 = .37014 + .00011 = .37025

Calculating antilogarithms

We can use a table of logarithms to calculate antilogarithms by using the table backwards

- For example, to find the antilogarithm of .37025
  - We find that .37025 lies between the entries
    - log 2.345 = .37014
    - log 2.346 = .37033
  - The difference between these entries is 19
  - .37025 is 11 more than the first entry
  - 11 is entry for 6 in the proportional parts for the difference of 19
- Therefore the antilog of .37025 is 2.3456

Definitions

- The integer part of the logarithm is called the "characteristic"
- The fractional part of the logarithm is called the "mantissa"

For numbers between 0 and 1 the logarithm is negative

- Generally the mantissa is made positive with a negative characteristic
  - A negative characteristic can be indicated by drawing a line above the characteristic
    - $\bar{1}$.69897 = (-1) + .69897 = log .5
  - Sometimes we will have to convert a negative mantissa to a positive mantissa by adding 1 to it and subtracting 1 from the characteristic

The characteristic tells where the decimal point goes

The mantissa tells what the digits are

Multiplication using logarithms

Let's multiply $12345 \times 54321 = n$

log 12345 = log ($10^4 \times 1.2345$) = 4 + log 1.2345

log 1.234 = .09132

log 1.235 = .09167 difference = 35

proportional part of 5 for a difference of 35 is 18

log 1.2345 = log 1.234 + 18

log 1.2345 = .09132 + 18 = .09150

log 12345 = 4.09150

log 54321 = log ($10^4 \times 5.4321$) = 4 + log 5.4321

log 5.432 = .73496

log 5.433 = .73504 difference = 8

proportional part of 1 for a difference of 8 is 1

log 5.4321 = log 5.432 + 1

log 5.4321 = .73496 + 1 = .73497

log 54321 = 4.73497

log 12345×54321 = 4.09150 + 4.73497 = 8.82647

log n = $10^8 \times$ antilog .82647

log 6.706 = .82646

log 6.707 = .82653 difference is 7

.82647 = .82646 + 1

1 is the proportional part of 1 for a difference of 7

log 6.7061 = .82647

$n = 10^8 \times 6.7061 = 670610000 \approx 670592745$