

Logarithms

In spherical trigonometry

there are many formulas that require multiplying two sines together, e.g., for a right spherical triangle

$$\sin b = \sin B \sin c$$

In the 1590's it was known (as the method of prosthaphaeresis) that by using the trigonometric identity this could be simplified to

$$\sin b = \sin B \sin c = [\cos(B - c) - \cos(B + c)] / 2$$

replacing a multiplication by an addition

This probably encouraged people to look for methods to replace multiplication with simpler operations

Also it was known that to multiply two numbers of the series

$$1, 2, 4, 8, 16, \dots \text{ or } 2^0, 2^1, 2^2, 2^3, 2^4, \dots$$

we just need to add the exponents of the two numbers

Napier

Napier (1550-1617) using geometrical reasoning published in 1614 a 56 page text and 90 pages of tables describing logarathms

His original tables were of logarathms of sines

The base of his logarithms were somewhat $1/e$ although there was a factor of 10^7 and the log of 1 was not 0

Logarithms to the base $e = 2.718281828459045\dots$ are known as Napierian logarithms or natural logarithms

Briggs

Henry Briggs obtained a copy of Napier's publication and became enthusiastic about logarithms

He met and had several discussions with Napier and improved them by

Making the log of 1 to be 0

Making 10 be the base of the logarithm (although integers were preferred in the tables so logs were multiplied by 10^{10})

Logarithms to the base 10 are also known as common logarithms or Briggsian logarithms

Basic operations with logarithms

The antilogarithm of n is the number whose logarithm is n

To multiply two numbers we add their logarithms (and take the antilogarithm)

$$\log ab = \log a + \log b$$

To divide two numbers we subtract their logarithms (and take the antilogarithm)

$$\log a/b = \log a - \log b$$

To take a square root we divide the logarithm by 2 (and take the antilogarithm)

$$\log \sqrt{a} = \frac{1}{2} \log a$$

To take an n th root we divide the logarithm by n (and take the antilogarithm)

$$\log \sqrt[n]{a} = (1/n) \log a$$

To raise to the n th power we multiply the logarithm by n (and take the antilogarithm)

$$\log a^n = n \log a$$

Using a table of logarithms

Modern tables of (base 10) logarithms are arranged to give logarithms of numbers in the range of 1 to 10

To get the logarithm of a number outside of this range we multiply by a power of ten (shift the decimal point) until the number is in the range of the table

We then add the number of places we shifted to the table value effectively multiplying by the proper power of 10

Remember that the logarithm of 10 is 1

If the table does not have exactly the number we are taking the logarithm we must interpolate between the neighboring values

For example, if we want the log of 2.3456 but the table only has the log of 2.345 and 2.346 we

take the difference between the log of 2.345 and the log of 2.346

divide this difference by 10

multiply by the 6

and add the result to the log of 2.345

Many tables have a list of "proportional parts" to help in this

$$\log 2.345 = .37014$$

$$\log 2.346 = .37033$$

The difference is 19

The "6" entry in the proportional part table of 19 is 11

$$\log \text{ of } 2.3456 = .37014 + .00011 = .37025$$

Calculating antilogarithms

We can use a table of logarithms to calculate antilogarithms by using the table backwards

For example, to find the antilogarithm of .37025

We find that .37025 lies between the entries

$$\log 2.345 = .37014$$

$$\log 2.346 = .37033$$

The difference between these entries is 19

.37025 is 11 more than the first entry

11 is entry for 6 in the proportional parts for the difference of 19

Therefore the antilog of .37025 is 2.3456

Definitions

The integer part of the logarithm is called the "characteristic"

The fractional part of the logarithm is called the "mantissa"

For numbers between 0 and 1 the logarithm is negative

Generally the mantissa is made positive with a negative characteristic

A negative characteristic can be indicated by drawing a line above the characteristic

$$\overline{1}.69897 = (-1) + .69897 = \log .5$$

Sometimes we will have to convert a negative mantissa to a positive mantissa by adding 1 to it and subtracting 1 from the characteristic

The characteristic tells where the decimal point goes

The mantissa tells what the digits are

Multiplication using logarithms

Let's multiply $12345 \times 54321 = n$

$$\log 12345 = \log (10^4 \times 1.2345) = 4 + \log 1.2345$$

$$\log 1.234 = .09132$$

$$\log 1.235 = .09167 \text{ difference} = 35$$

proportional part of 5 for a difference of 35 is 18

$$\log 1.2345 = \log 1.234 + 18$$

$$\log 1.2345 = .09132 + 18 = .09150$$

$$\log 12345 = 4.09150$$

$$\log 54321 = \log (10^4 \times 5.4321) = 4 + \log 5.4321$$

$$\log 5.432 = .73496$$

$$\log 5.433 = .73504 \text{ difference} = 8$$

proportional part of 1 for a difference of 8 is 1

$$\log 5.4321 = \log 5.432 + 1$$

$$\log 5.4321 = .73496 + 1 = .73497$$

$$\log 54321 = 4.73497$$

$$\log 12345 \times 54321 = 4.09150 + 4.73497 = 8.82647$$

$$\log n = 10^8 \times \text{antilog } .82647$$

$$\log 6.706 = .82646$$

$$\log 6.707 = .82653 \text{ difference is } 7$$

$$.82647 = .82646 + 1$$

1 is the proportional part of 1 for a difference of 7

$$\log 6.7061 = .82647$$

$$n = 10^8 \times 6.7061 = 670610000 \approx 670592745$$