

## Recurrent Neural Networks

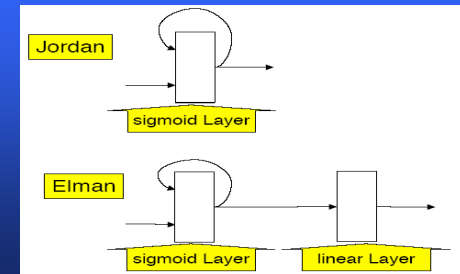
### Topic 7

Note: lecture notes by Bob Keller (Harvey Mudd College, CA) are used

Main idea: include feedback from neuron output to input into NN models

- A recurrent network is one in which there is feedback from a neuron's output to its input.
- Various models exist:
  - Jordan Network (feedback from net output to input)
  - Elman Network ("partially recurrent": feedback from internal state output to input)
  - Hopfield Network

## Jordan vs. Elman Networks



## Elman networks



### ■ References:

Elman, J. L. (1990). Finding structure in time. *Cognitive Science*, 14:179-211.

Neural Networks: Automata and Formal Models of Computation, Mikel L. Forcada,

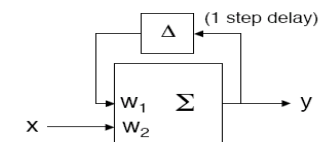
<http://www.dlsi.ua.es/~mlf/naafmc/pbook/pbook.html>

## How to train Elman networks?

- One way:
  - Initialize the state values to nominal.
  - Repeat
    - Simulate one step of the network.
    - Compute the actual output.
    - Backpropagate the error.
    - Adjust the weights.
    - Compute the next state.
  - Until the error is sufficiently low.

## Training feedback weights

### ● Elman Schematic



$$y(k) = w_1 y(k-1) + w_2 x(k)$$

## Training feedback weights

$$y(k) = w_1 y(k-1) + w_2 x(k)$$

$$\frac{\partial}{\partial w_2} y(k) = x(k)$$

$$\frac{\partial}{\partial w_1} y(k)$$

$$= \frac{\partial}{\partial w_1} w_1 y(k-1)$$

$$= y(k-1) + w_1 \frac{\partial}{\partial w_1} y(k-1) \quad \text{[derivative of a product]}$$

$$= y(k-1) + y(k-2) + w_1 \frac{\partial}{\partial w_1} y(k-2) \quad \text{[a recurrence]}$$

$$= y(k-1) + y(k-2) + y(k-3) + \dots$$

Conclusion: The sensitivities will depend on all previous values of the output.

## Demos of Elman Networks

Two demos:

- Matlab appelm1
- NAS demo 11.2

## Other possible ways to train

- BPPT (Backpropagation Through Time) would be another way:
  - unroll the network some large number of levels,
  - backpropagate,
  - average the weight changes over the **unrolled** stages to get a single set of weight changes