Sampling Edge Covers in 3-Regular Graphs

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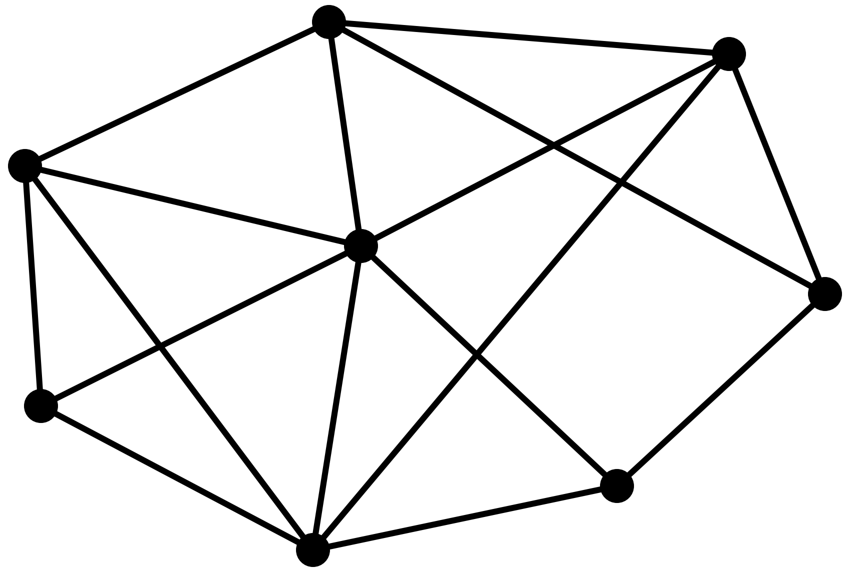
Parsing the title: Edge Covers

Given: an undirected graph G=(V,E)

An **edge cover** of G: a set of edges covering all vertices

Example:



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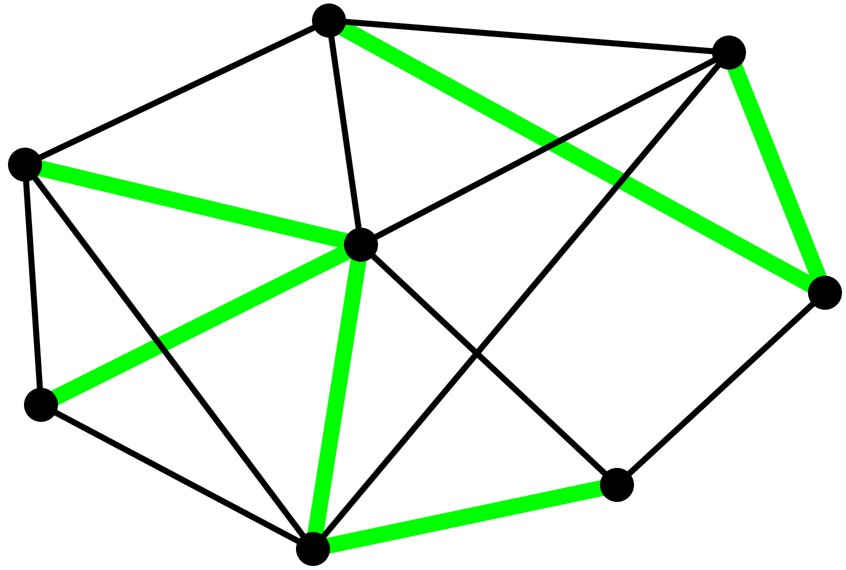

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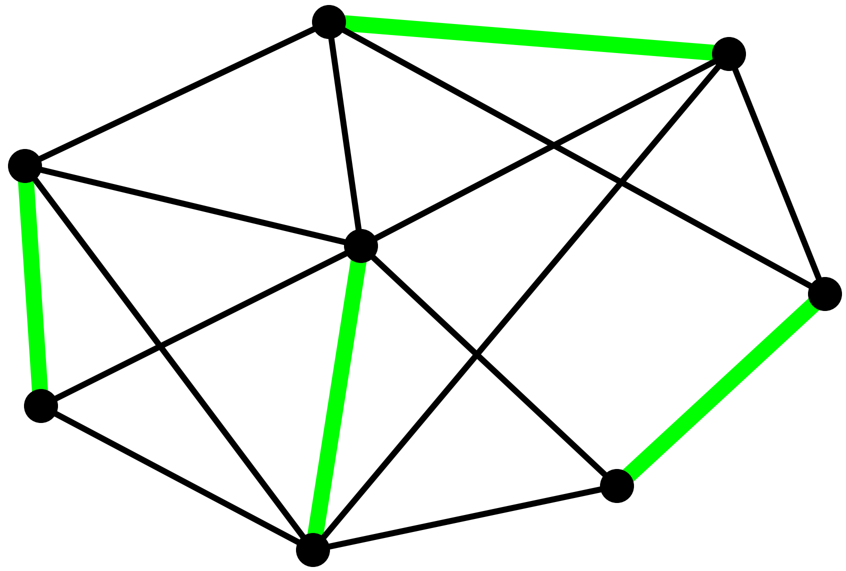


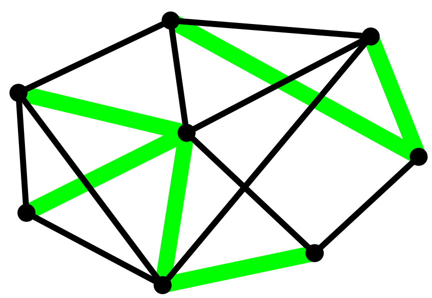
another possible edge cover

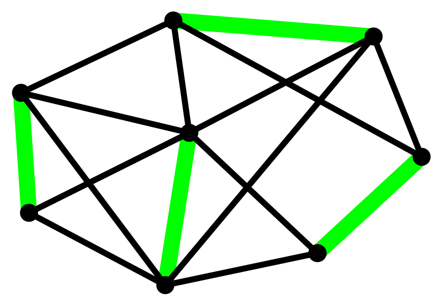
Parsing the title: Sampling Edge Covers

The problem: given an undirected graph G, return **a uniformly random sample from all edge covers** of G.

- let Ω be the set of all edge covers of G

- uniformly random: every edge cover is sampled with probability $1/|\Omega|$





Motivation

Edge covers are related to other problems:

- (perfect) matchings of a graph G
- graphs with a given degree sequence (the bipartite case also known as binary or 0/1 contingency tables)
- vertex covers, independent sets

These problems are well-studied sampling problems, with applications, e.g., in statistical physics, biology, and various areas of CS.

Motivation & Related Work

Sampling version of the match./deg.seq. problem is open for:

- perfect matchings for arbitrary graphs
- graphs with a given (arbitrary) degree sequence

Most relevant related works – known poly-time samplers:

- matchings:
  - all in arbitrary graphs, perfect in dense graphs: [Jerrum-Sinclair '89]
  - perfect in bipartite graphs: [Jerrum-Sinclair-Vigoda '04]
- graphs with a given degree sequence:
  - regular seq., bipartite graphs: [Kannan-Tetali-Vempala '89]
  - arbitrary seq., dense or bipartite graphs: [JS '89, JSV '04]
  - regular sequence, non-bipartite graphs: [Cooper-Dyer-Greenhill '07]

Motivation & Related Work

Sampling version of the match./deg.seq. problem is **open** for:
- perfect matchings for arbitrary graphs
- graphs with a given (arbitrary) degree sequence

Our motivation: issues arising when attempting to work with these open problems are similar to issues arising with analyzing edge covers

Note:

|min vertex cover| + |max indep. set| = n
|min edge cover| + |max matching| = n

where n = number of vertices.

However, the above relationship cannot be directly used for deriving sampling algorithms for edge covers from matchings, or vice versa.

Motivation & Related Work

Graphs with bounded degree are of interest in statistical physics – most relevant related works:

- perfect matchings: [Luby-Randall-Sinclair '01]
- independent sets: [Luby-Vigoda '99], [Dyer-Greenhill '00], [Vigoda '01]

Note: These works (except DG) use a Glauber-dynamics-type Markov chain approach.

Our contributions

Thm: There is a fully polynomial approximate sampler for edge covers of any graph with maximum degree 3.

Where:

- fully polynomial approximate sampler (fpas) = a randomized algo returning a sample from a distribution at most ϵ-distant from the uniform distribution, running time is polynomial in the size of the input and $1/\epsilon$

- fully polynomial randomized approximation scheme (fpras) = a randomized $(1+\epsilon)$-approximation algo with success probability $1-\eta$, running time is polynomial in the size of the input, $1/\epsilon$, and $1/\log \eta$

Our contributions

Thm: There is a fully polynomial approximate sampler for edge covers of any graph with maximum degree 3.

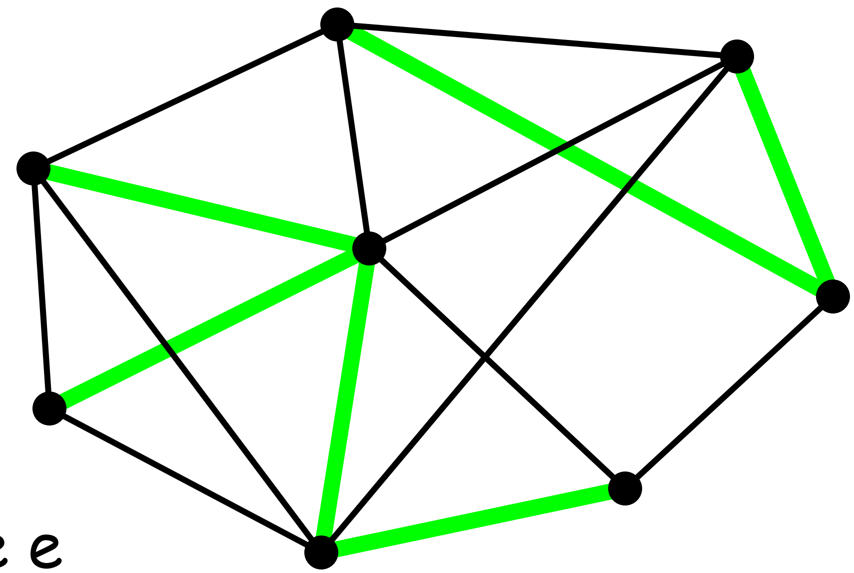
In the extended version:

Cor: There is an fpras for counting all edge covers of graphs with maximum degree 3.

Thm: There is a fully polynomial approximate sampler (and fpras for counting) all 1-2-edge covers (i.e., path-cycle covers) for n/3-dense graphs.

A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.



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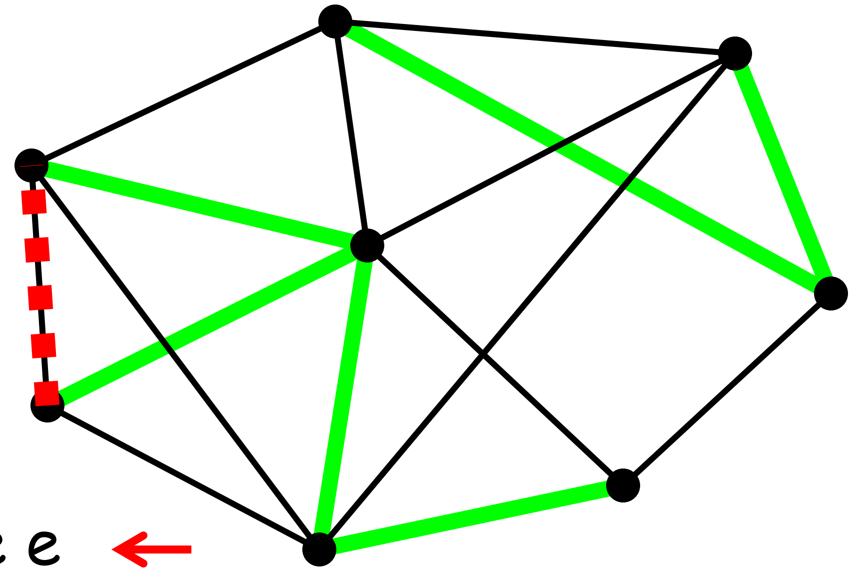

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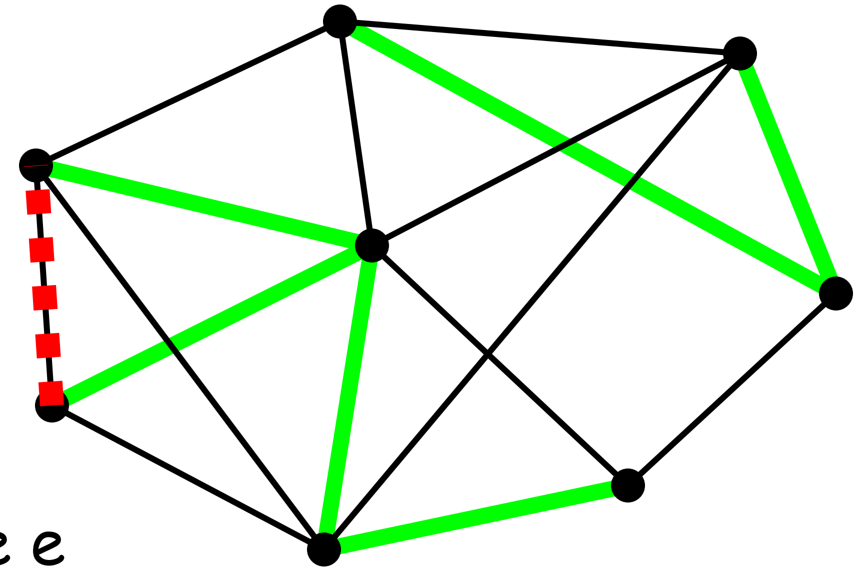

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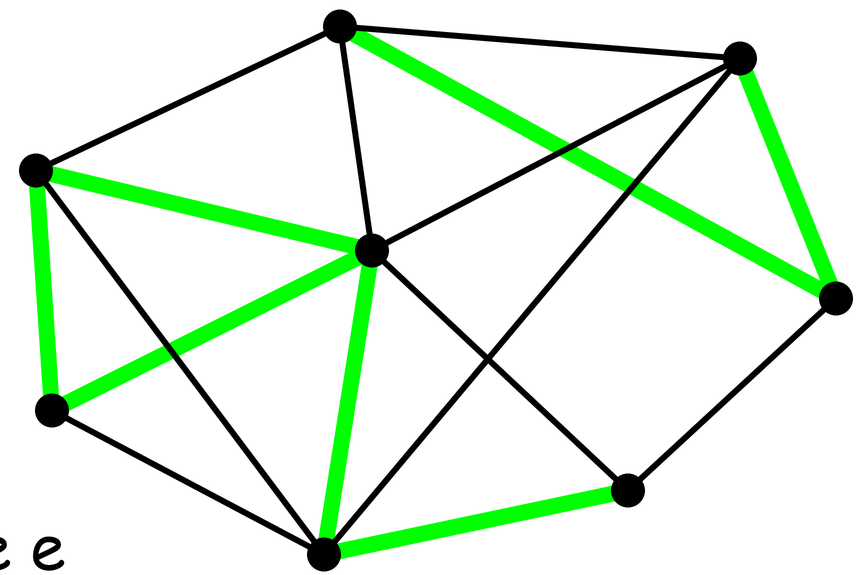

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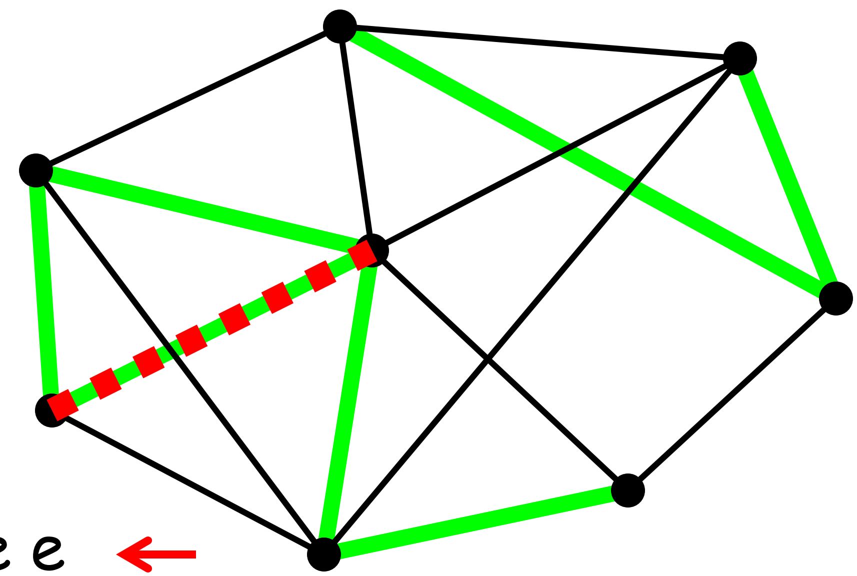

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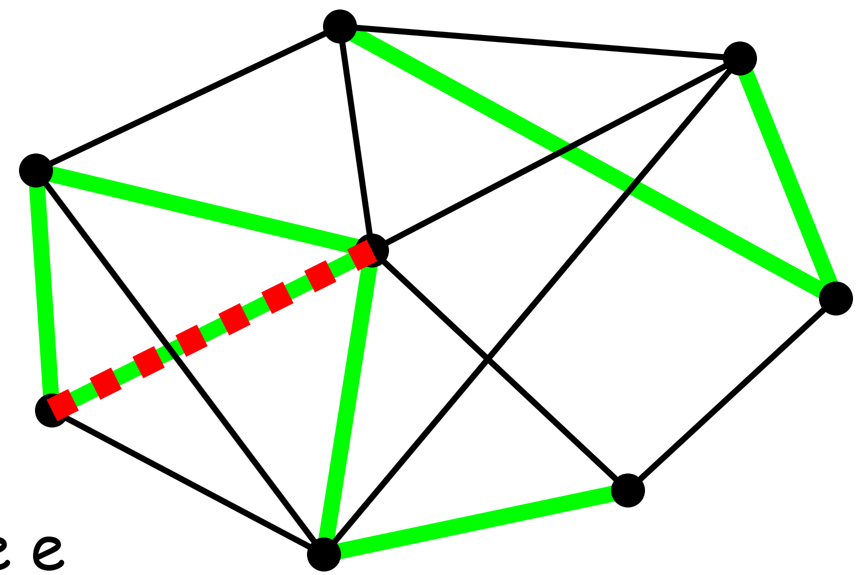

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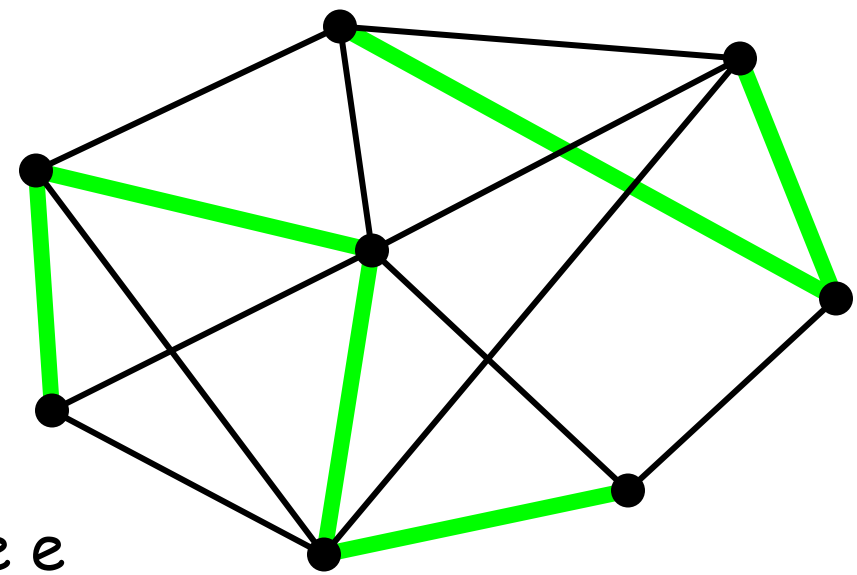

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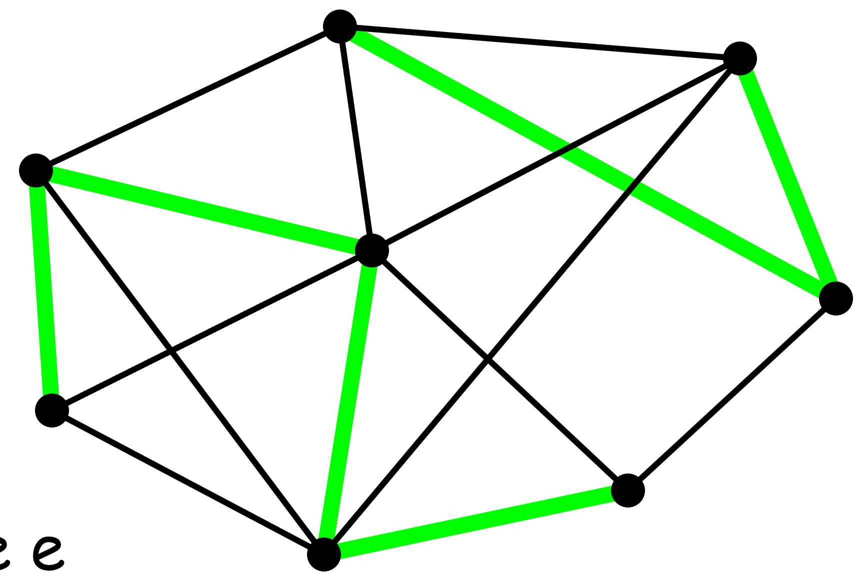

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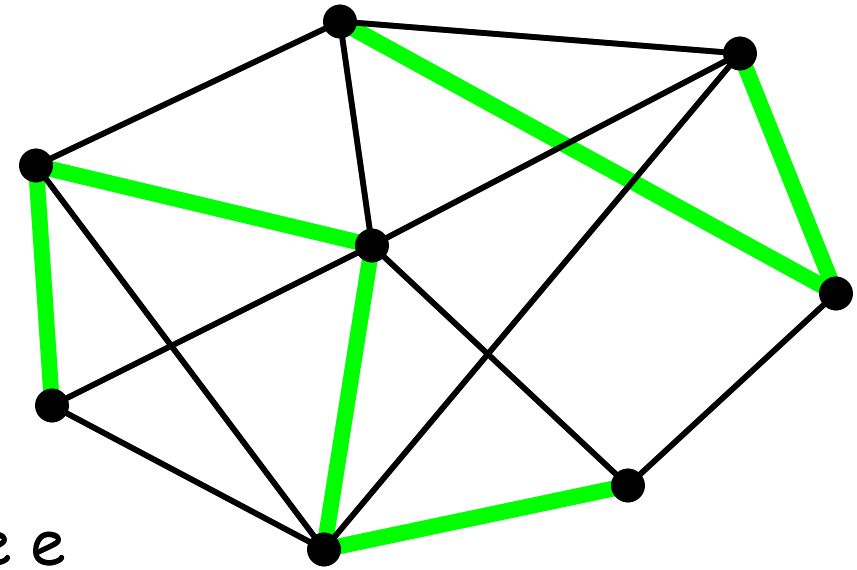

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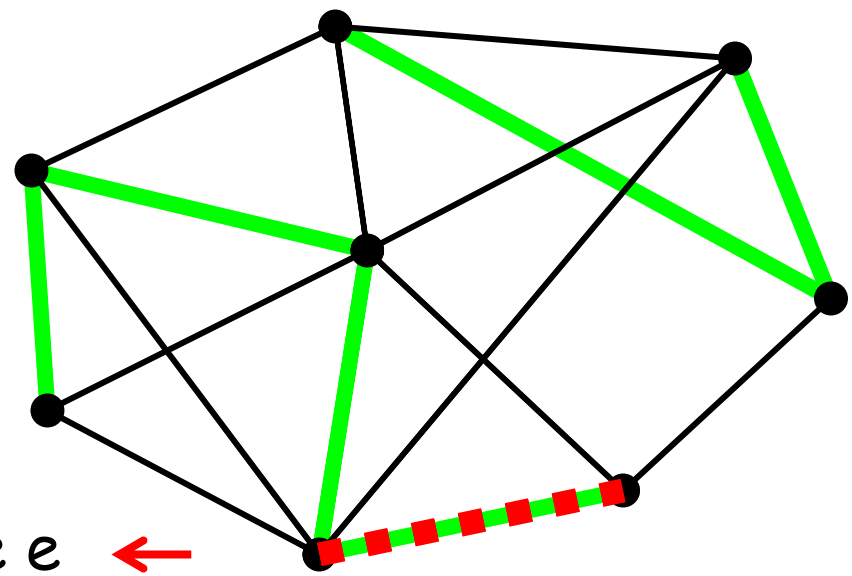

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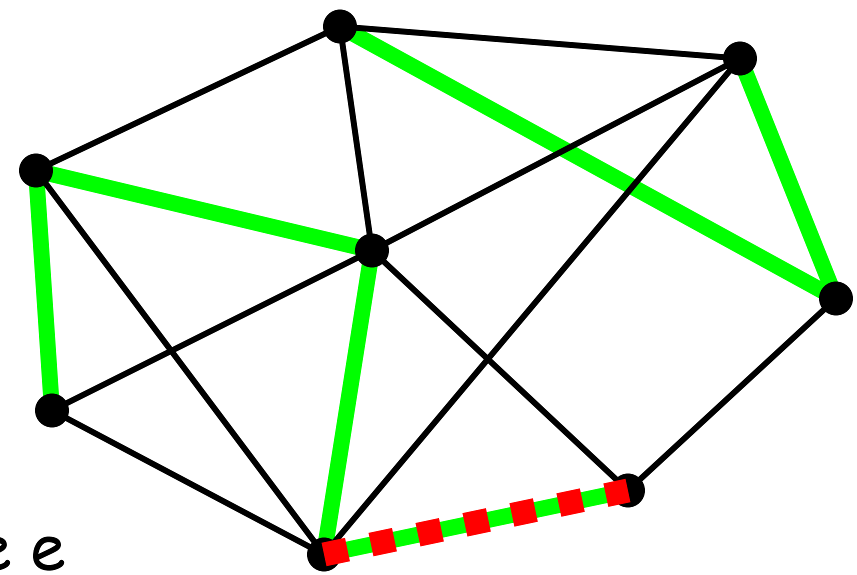

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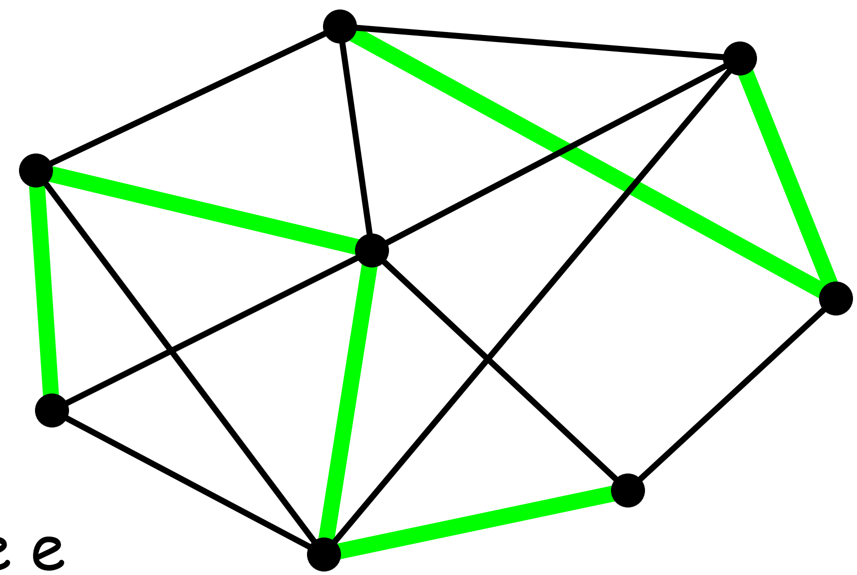

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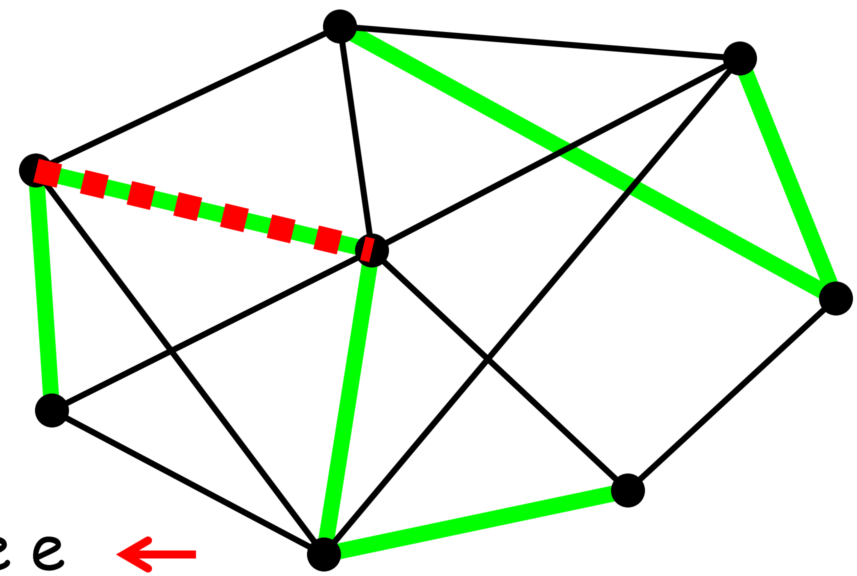

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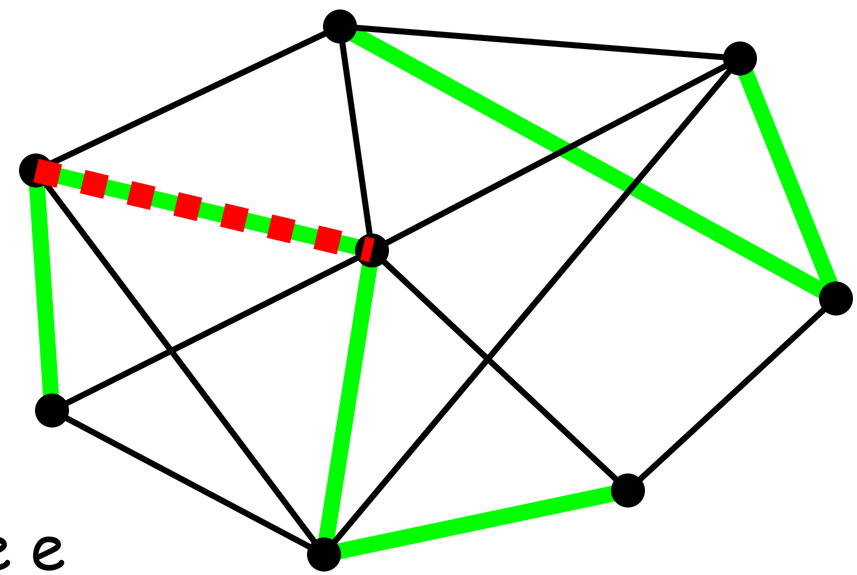

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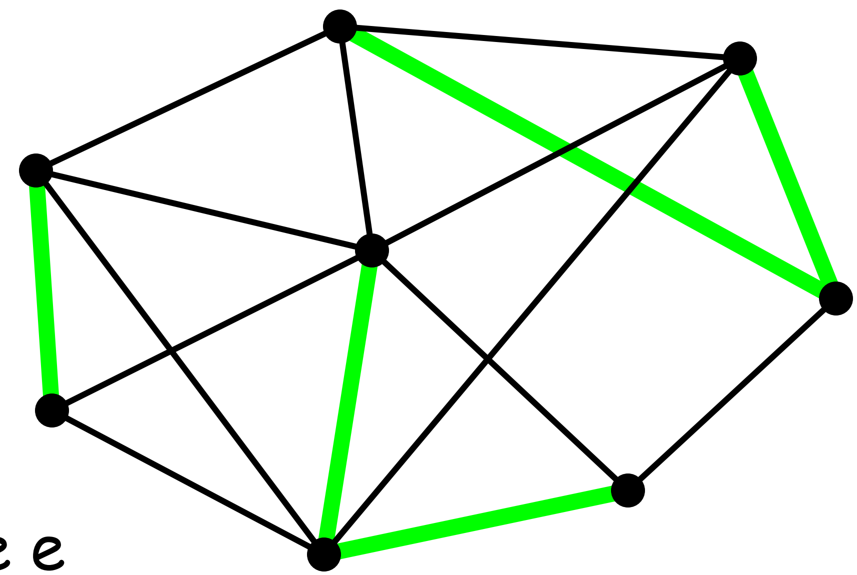

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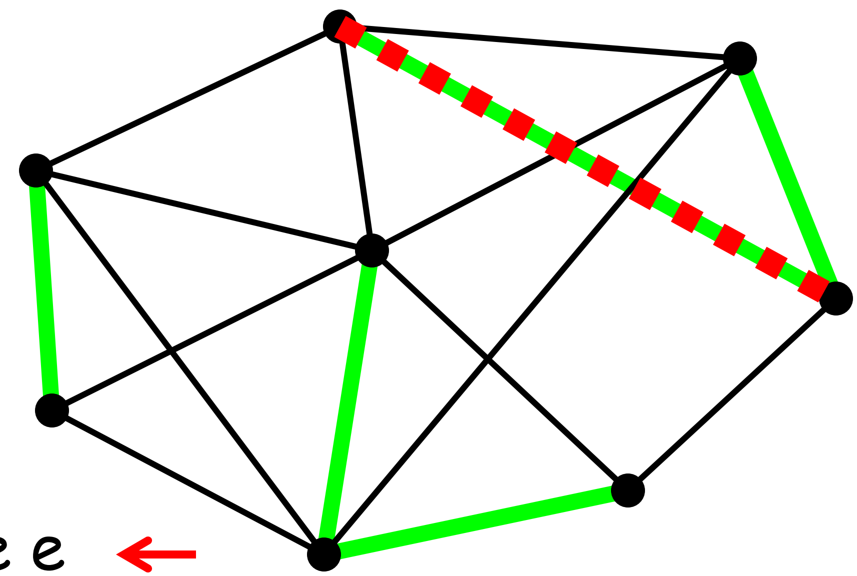

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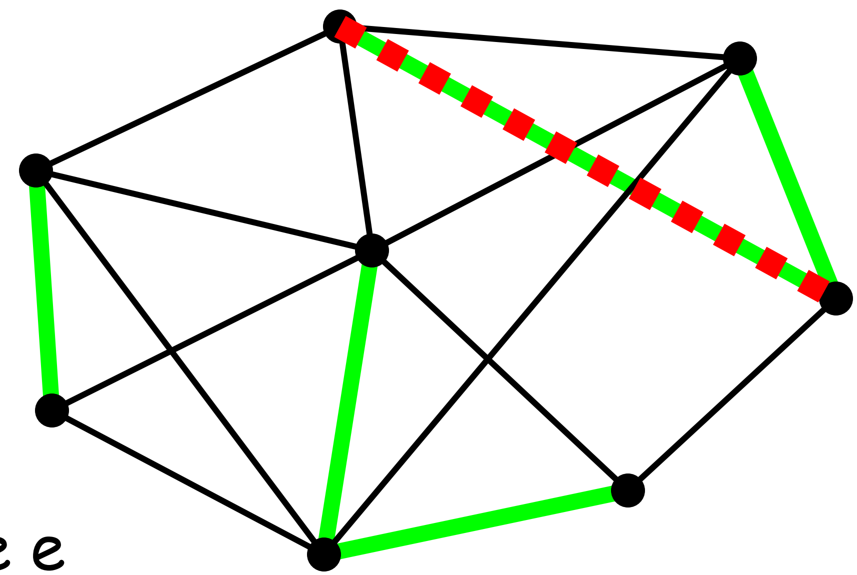

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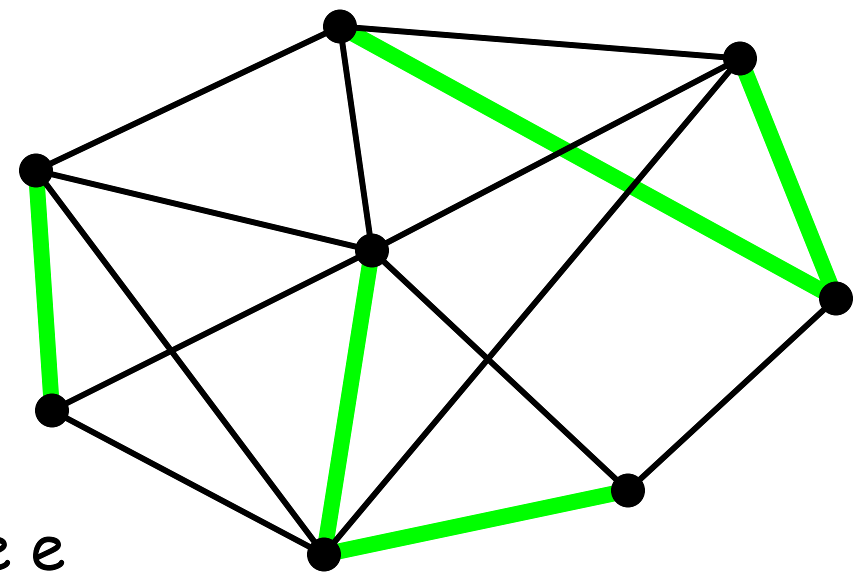

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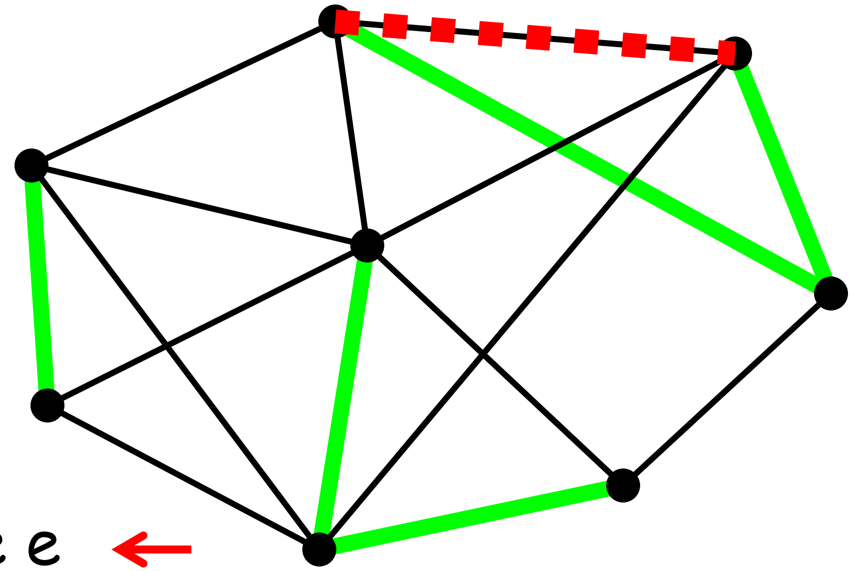

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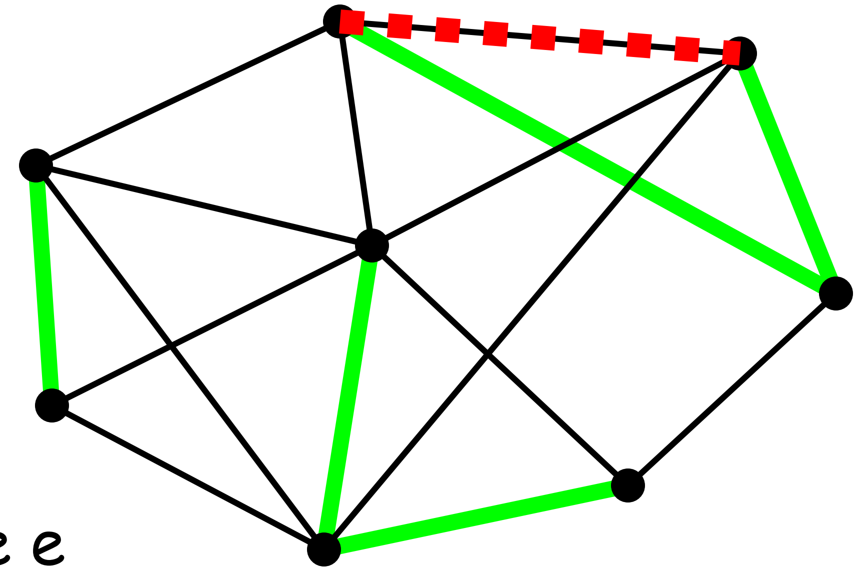

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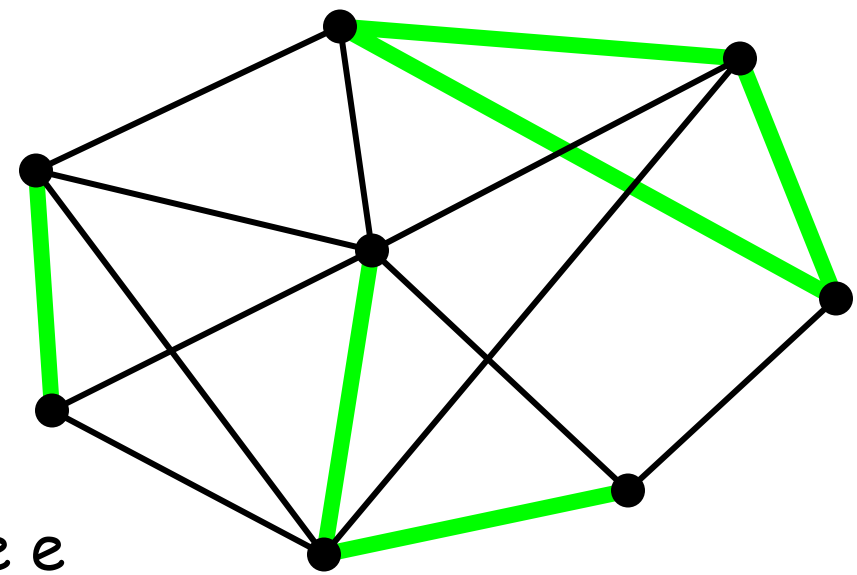

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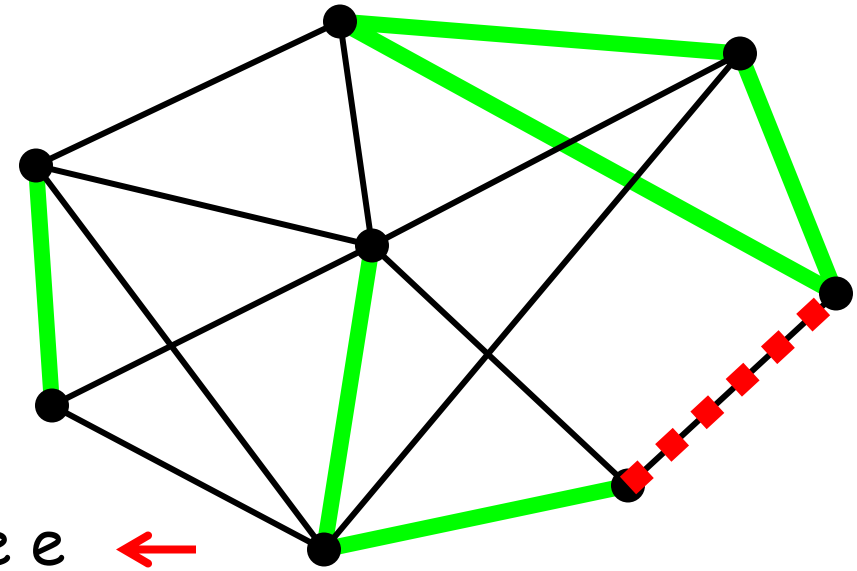

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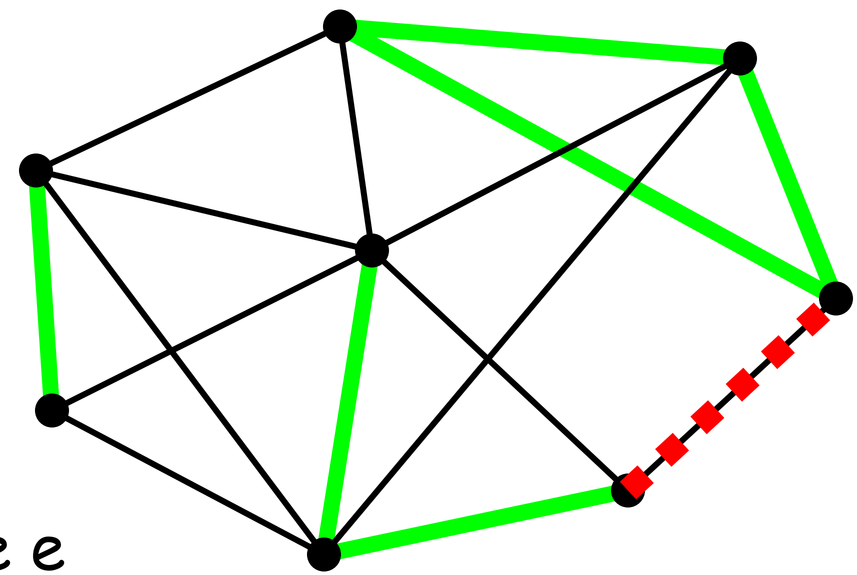

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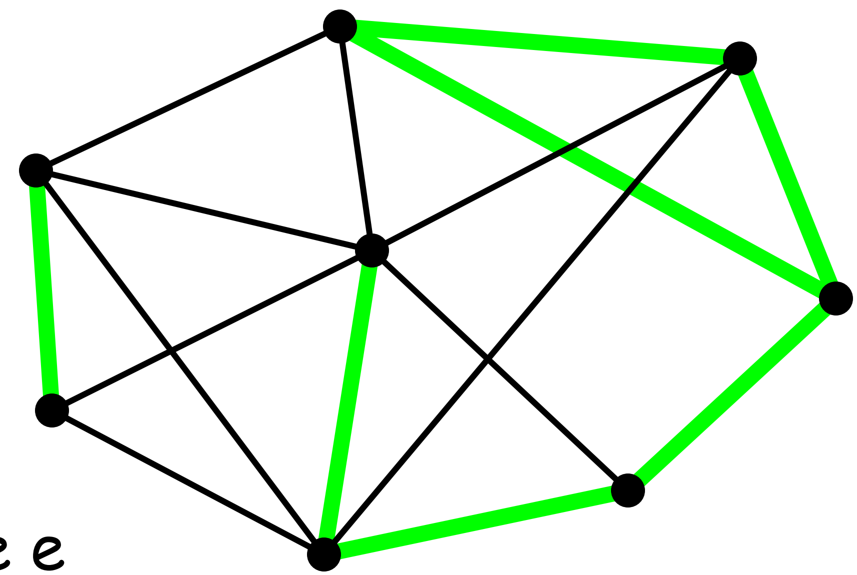

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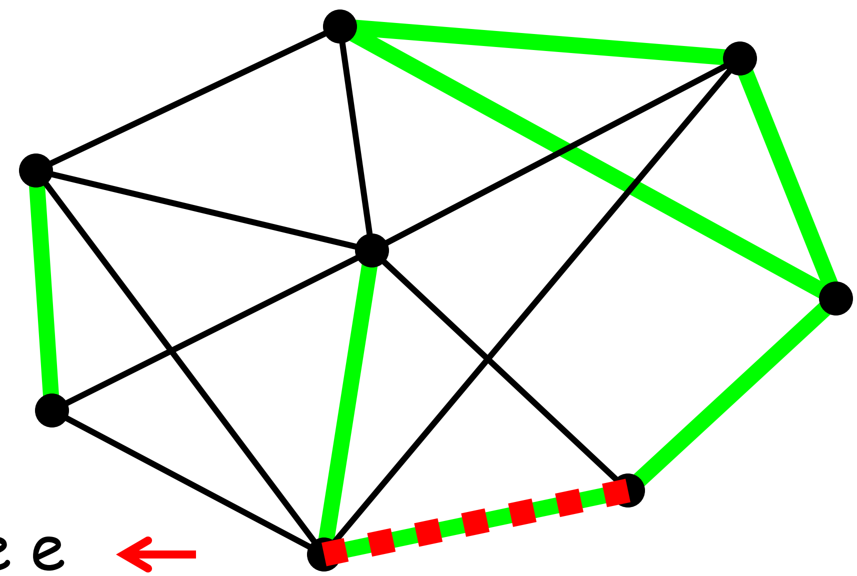

The Markov chain:

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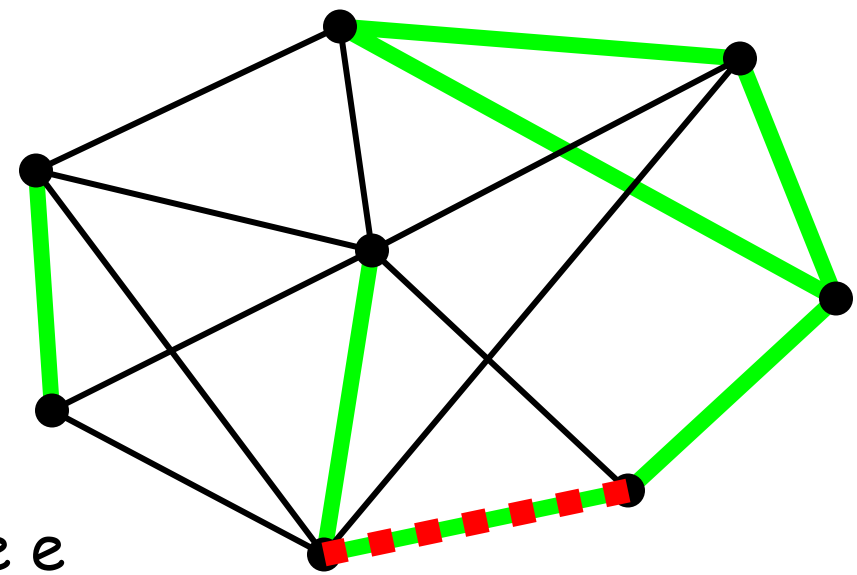

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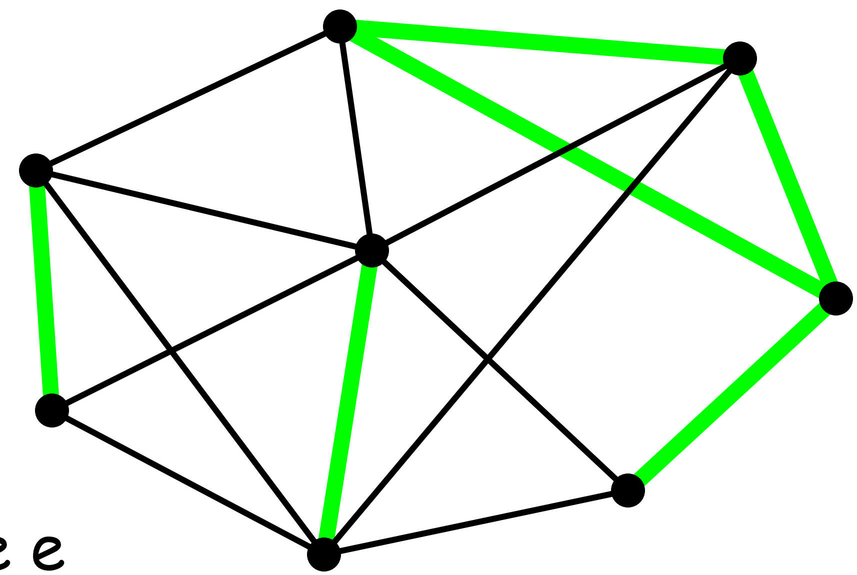

The Markov chain:

- from edge cover X_i:
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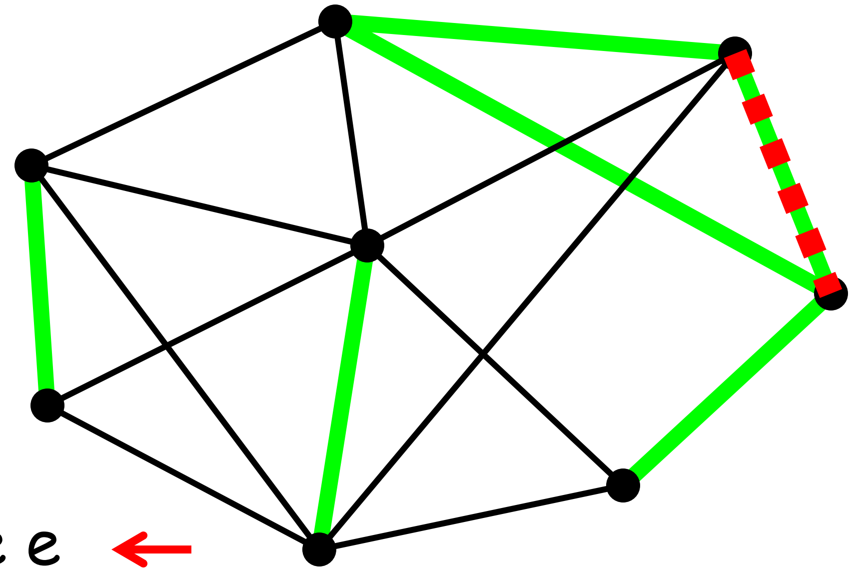

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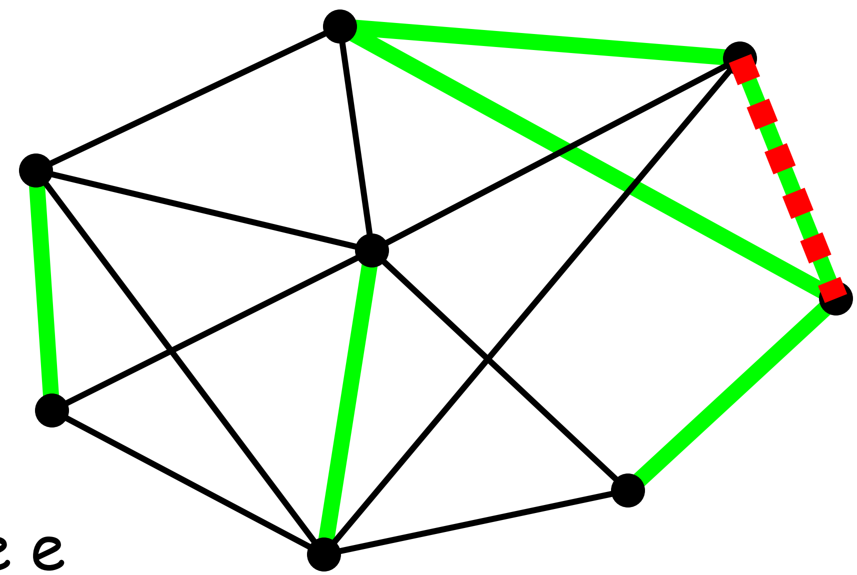

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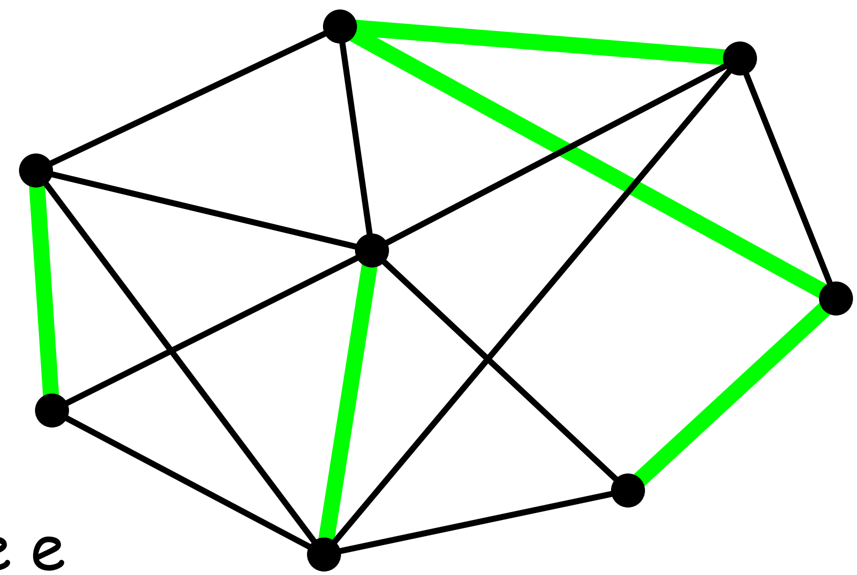

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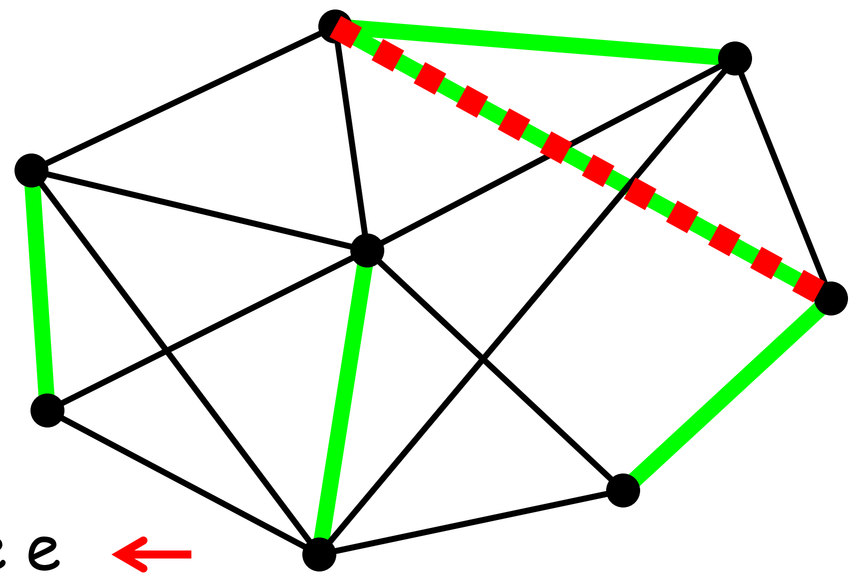

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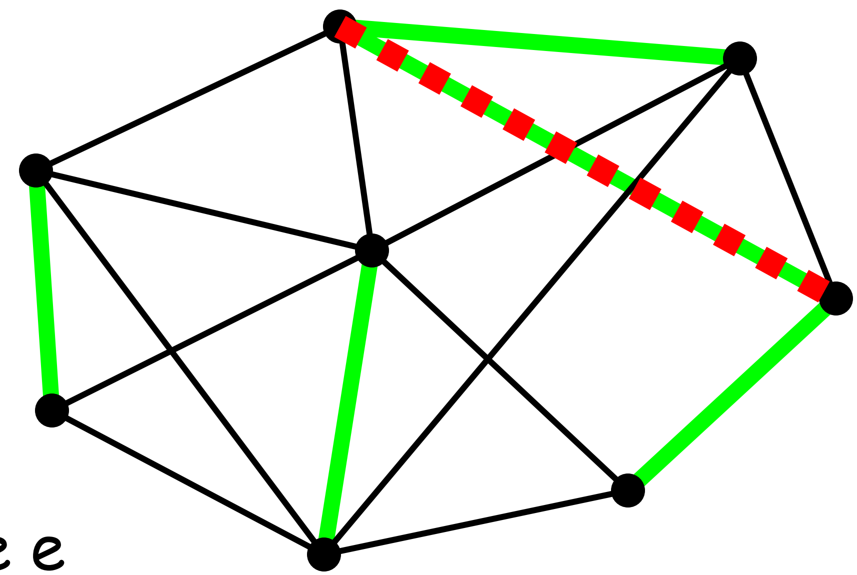

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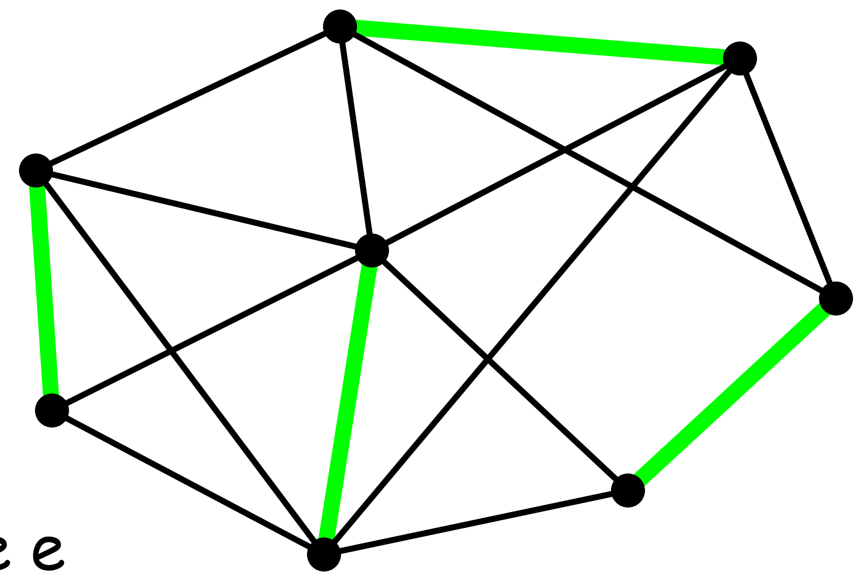

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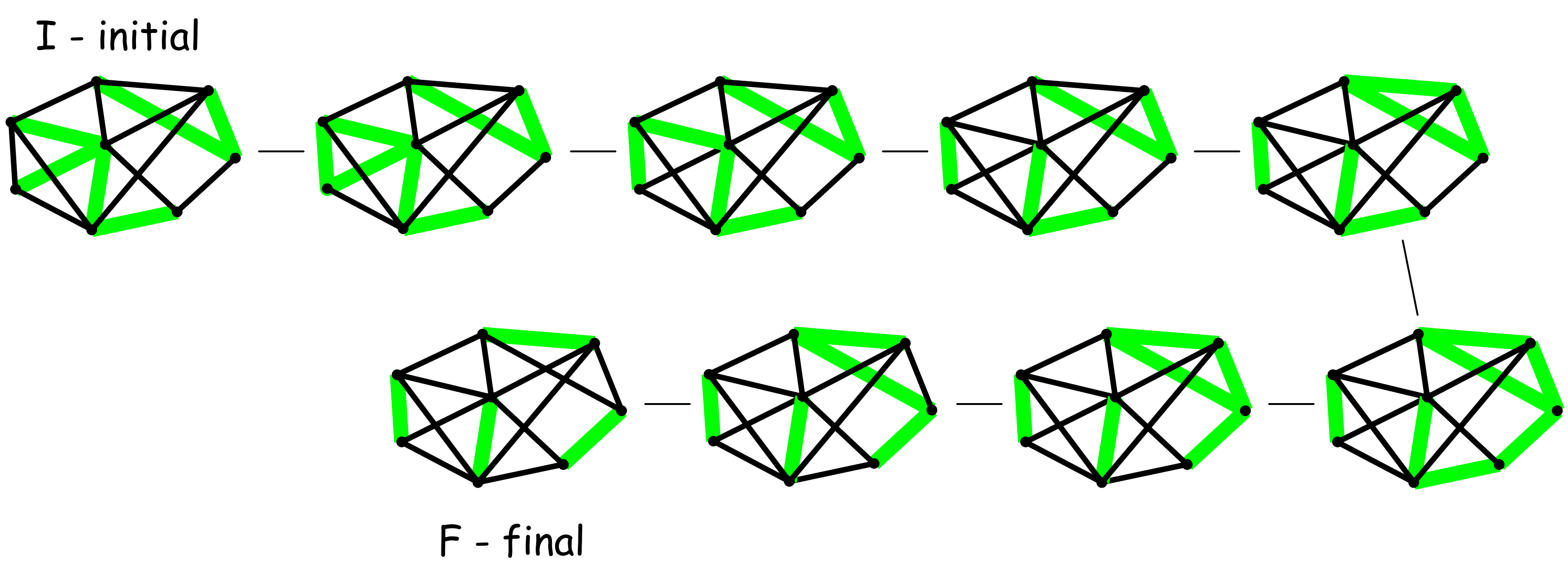
Proving rapid mixing: Canonical Flow

Consider the graph of all edge covers (all configurations), edges connect endpoints distant at one step of the MC.

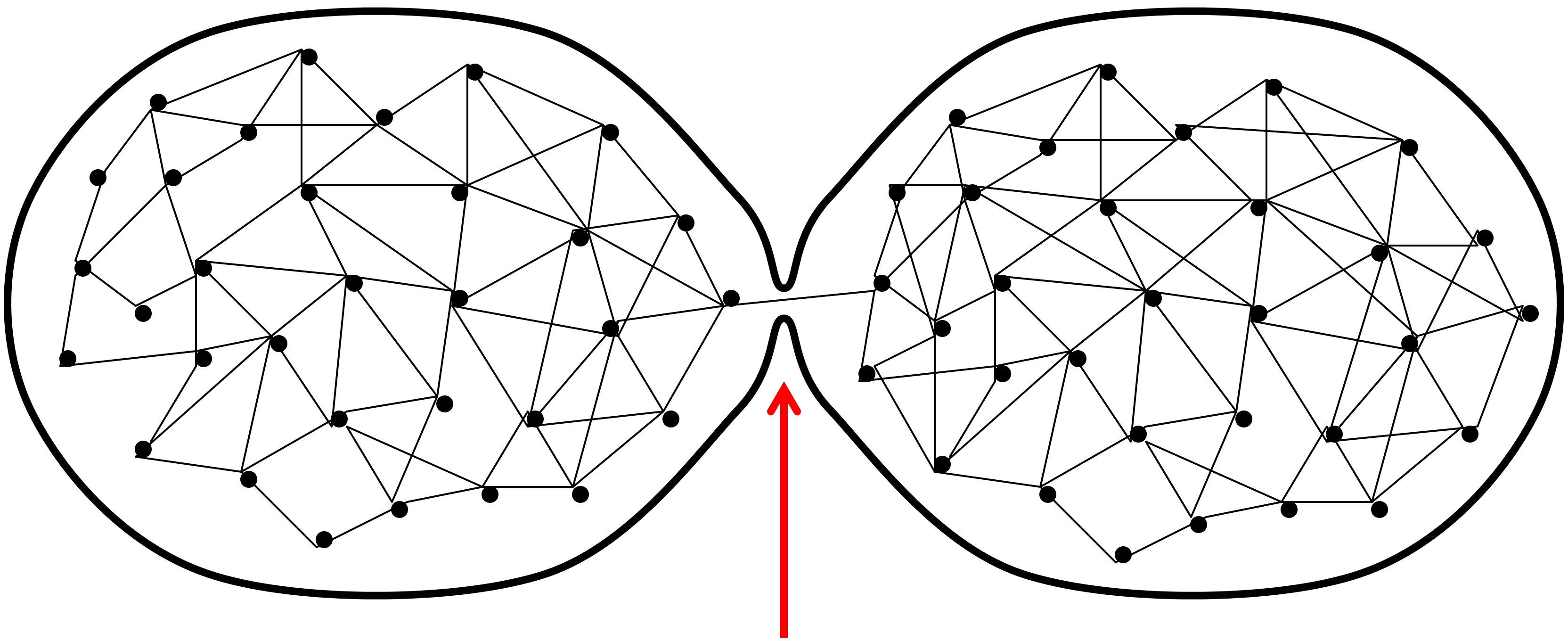
Canonical path:
for two configs I, F, a sequence of configs from I to F, uniquely given by a canonical order (of edges, etc).



Proving rapid mixing: Canonical Flow

Why canonical paths ?

Consider the graph of all configurations ($|\Omega|$ vertices) and all I-F canonical paths:



Transition used by $\sim |\Omega|^2/4$ paths -> MC not mixing rapidly.

Proving rapid mixing: Canonical Flow

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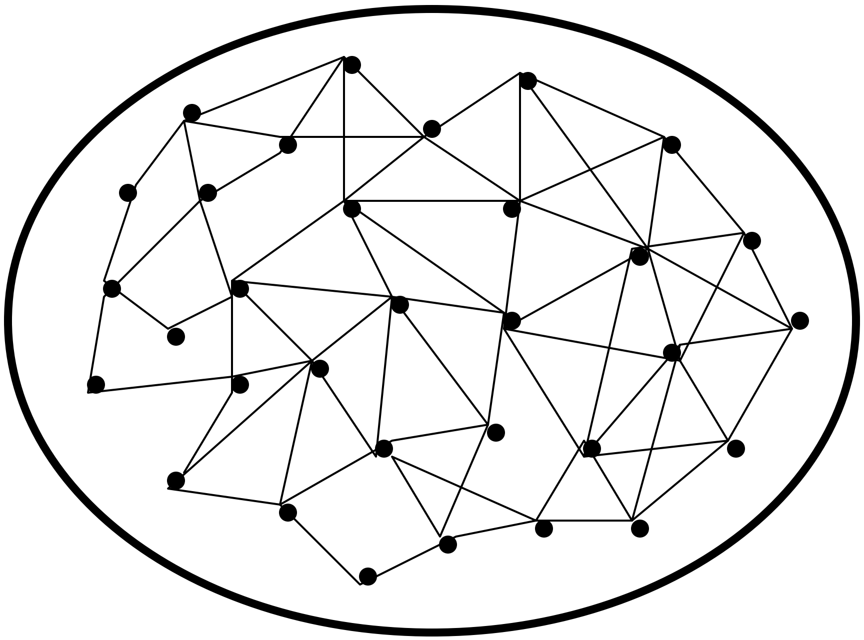


If all transitions used by poly number of paths times $|\Omega|$, then MC rapidly mixing.

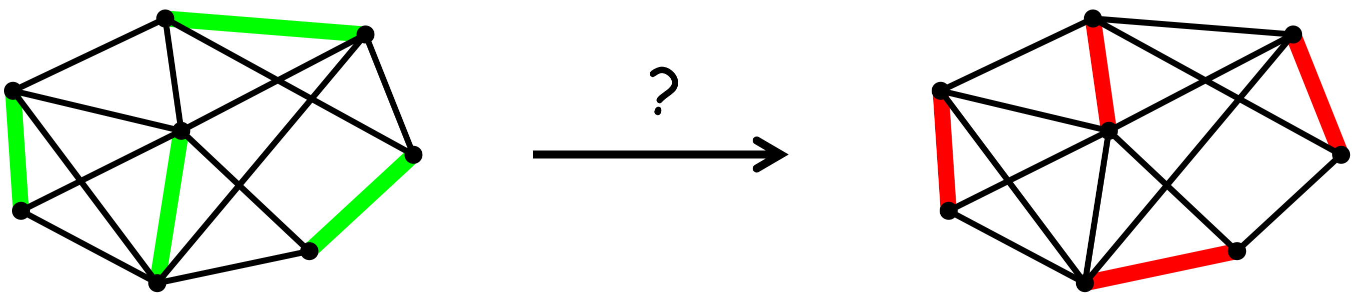
[Diaconis-Stroock '91, Sinclair '92]

Note: the number of vertices is often exponential (in the input size)

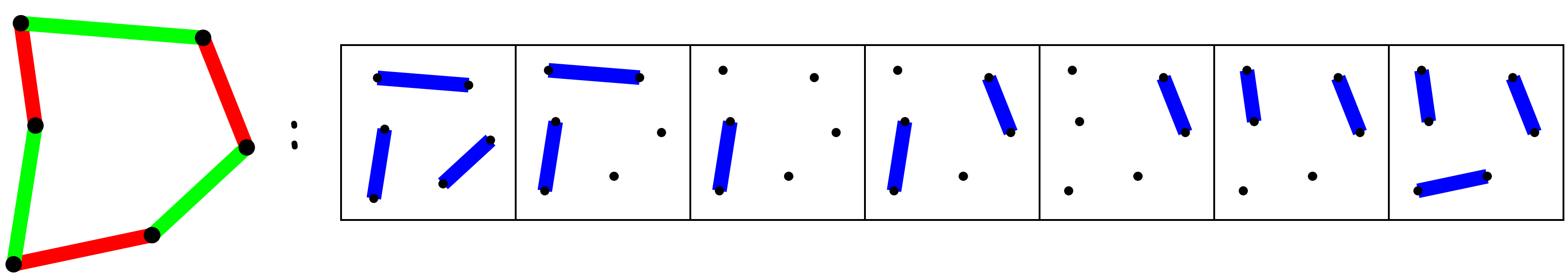
Note: canonical flow = unit flow from I to F through several I-F paths.

Canonical paths for matchings

Let I, F be two different matchings of a graph G:



- take symmetric difference
- order cycles/paths (using a predetermined canonical order)
- deal with each component



[Jerrum-Sinclair '89]

Canonical paths for matchings

Why these specific canonical paths ?

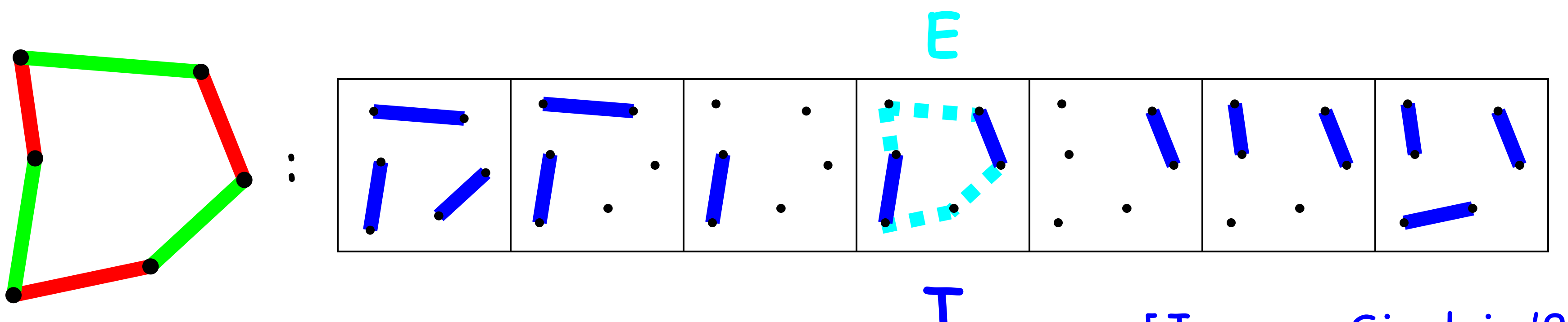
- let T be any transition on the path from I to F

- we need E = I⊕F⊕T to be close to a valid matching

- given T, E "encodes" (uniquely determines) I and F

- thus the number of paths through T is |Ω| times a polynomial in the input size

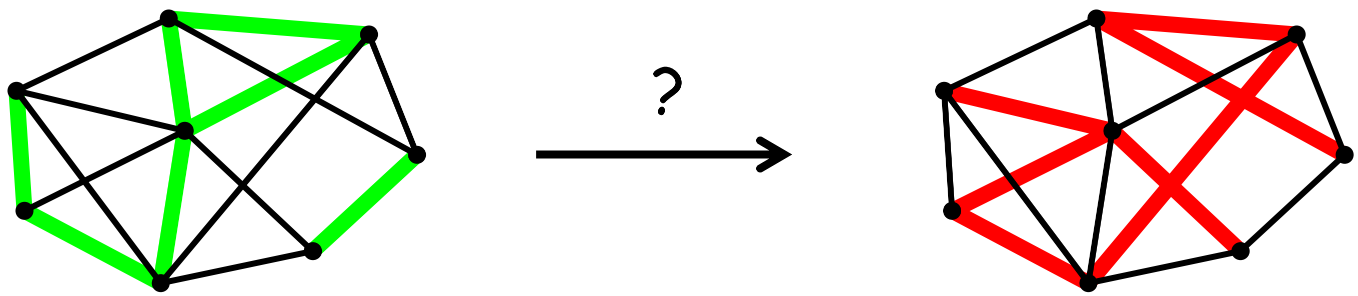
- implies small congestion and rapid mixing



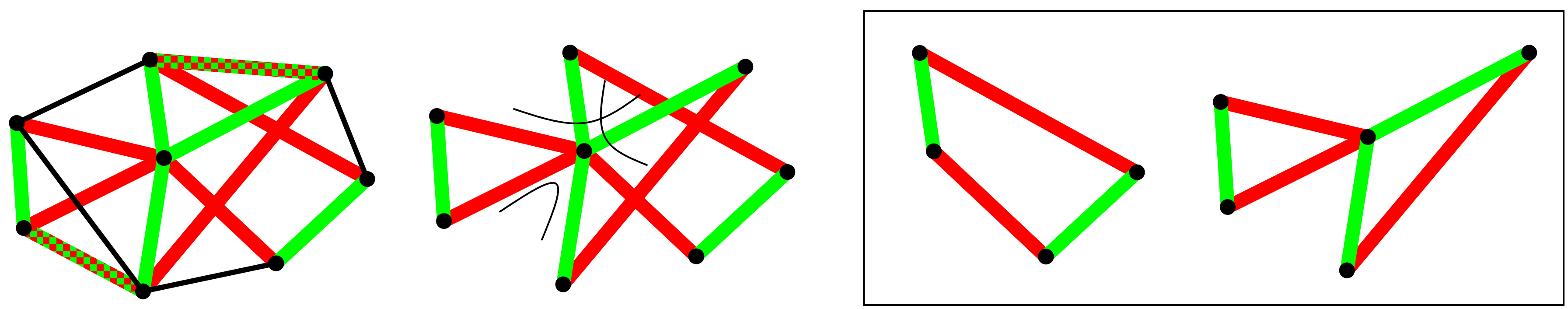
[Jerrum-Sinclair '89]

Canonical flow for graphs w. given deg.seq.

Consider I, F:



- take sym. diff. and an alternating cycle decomposition
- decomposition: **pair** green with red edges at every vertex



$I \cup F$

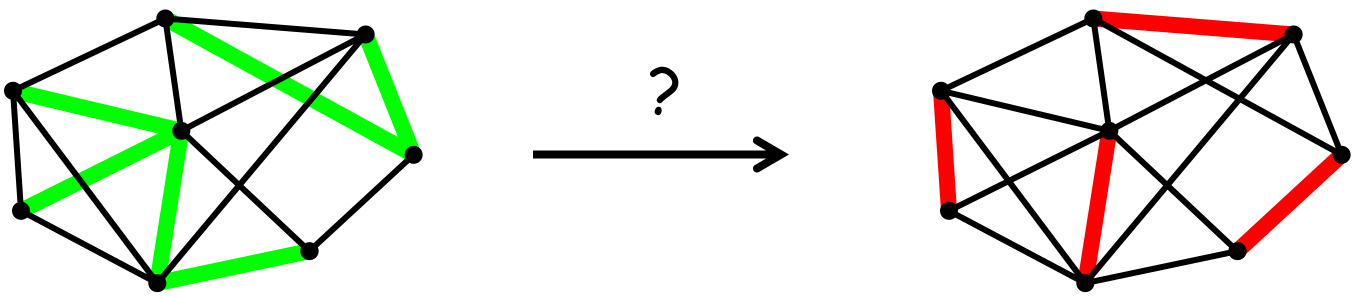
$I \oplus F$ and
a pairing at the middle vertex

decomposition

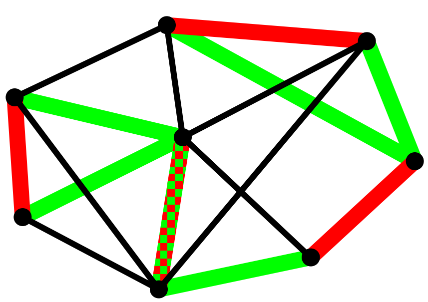
[Cooper-Dyer-Greenhill '07]

Canonical flow for edge covers

Consider I, F:



- take sym. diff. and a cycle/path-decomposition, not necessarily alternating, as follows:



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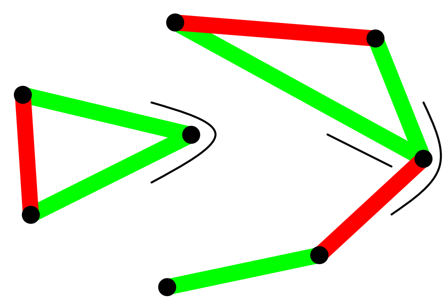

$I \oplus F$ and
a pairing at the middle and right vertices

pair green with red edges at every vertex, if left-over edges, pair the same color together

[this paper]

Canonical flow for edge covers

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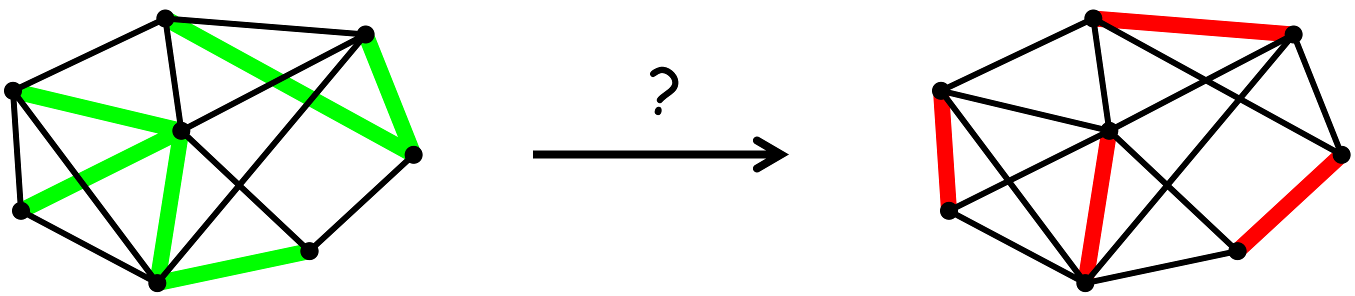

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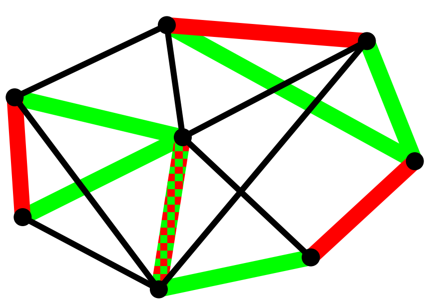
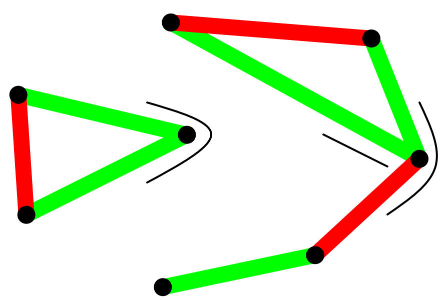
The catch: prove it works ☺



I ∪ F



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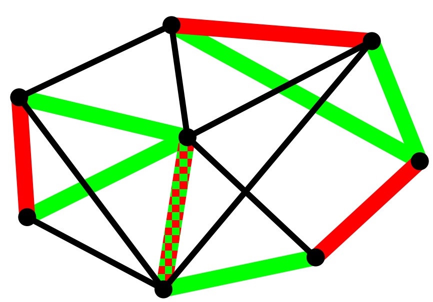
Canonical flow for edge covers

A little bit about the proof:

- alternation is critical to show that I->F moves only through valid configurations

- with degrees $\leq$ 3 we succeeded to show valid canonical flow with non-alternating decomposition

- encoding is "close" to an edge cover



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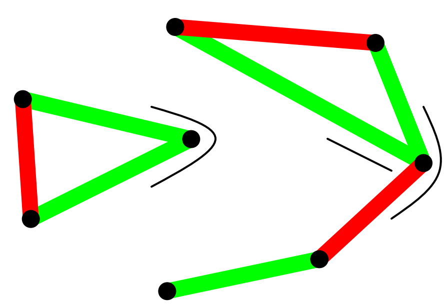

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Open problems

- edge covers for
  - constant degree graphs
  - bipartite graphs
  - general graphs (implications for matchings ?)
- minimum edge covers ? weighted edge covers ?
- non-alternating decomposition for other problems ?